Linear Programming Gasoline Blending

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INTRODUCTION

The introduction of linear programming (LP) has produced remarkable benefits in many industries — notably those that involve blending materials to manufacture a finished product, such as meat packing, ice cream blending, cotton blending, animal feed mixing, and primary metals alloying. Almost from its inception LP has been employed in the refining segment of the petroleum industry, and most large refineries currently use LP to plan daily and long-range operations. Considerable savings have been reported by the use of LP in the petroleum industry. (See reference 1 in the Bibliography.)

The purpose of this manual is to demonstrate the application of LP to the blending of gasoline, a process which, because it involves complex quality control, is particularly responsive to LP techniques. The immediate and most obvious LP results enable the gasoline producer (whether a refinery operator or a jobber) to minimize the cost of gasoline and the frequency of off-compositions and permit him to purchase and sell most economically.

The basis of the LP technique is the formulation of a mathematical model of the allocation problem. For problems of any practical size, this model is entered into a computer, and the computer LP system rapidly calculates the optimal (least-cost or maximum profit) solution. The system may also produce reports which indicate the effect on the optimal solutions of possible changes in the given prices, availabilities, product specifications, etc.

Contrary to popular belief, little mathematical knowledge or skill is required to formulate an LP model. Nor do the operation of the computer and the analysis of computer results require any advanced technical skill. Linear programming requires nothing more than the expression of all the factors in the process — blend specifications, component availabilities, costs — in the form of simple linear equations (or inequalities). The general principles of linear programming are discussed in the IBM data processing application manual An Introduction to Linear Programming (E20-8171), which should be read in conjunction with this manual.

To demonstrate the methods and advantages of LP in gasoline blending, we will develop an LP model for the solution of a typical (though simplified) production problem. This model will be solved by the IBM 1620/1311 Linear Programming System (1620-CO-04X); with minor modifications, it can be solved by any of IBM's LP systems. After establishing the basic technique for model formulation, we will describe a larger general model which incorporates a number of more sophisticated methods for linearizing essentially nonlinear processes. We may note that the techniques described in this manual may serve as a guide for blending not only gasoline but other products as well.

PROBLEM PROFILE AND ECONOMICS

The basic gasoline blending process involves the mixing of a number of available products, having different costs and physical characteristics, in order to achieve a specified gasoline quality. Further, additives, such as tetraethyl lead, are also used to control the product octane. The obvious economic problem is to determine the blend of components and the tetraethyl lead level which will produce the specified gasoline at least cost or maximum profit. The high volume typically involved in gasoline blending argues powerfully for the use of LP, since even slight per-unit savings will produce additional profit on a scale that amply justifies computer cost.

Manual determination of a maximum-profit blend is extremely difficult and laborious. To begin with, the complex interrelation among the qualities of the several components in the blends to be produced must be considered. Fluctuations in the prices and availabilities of components further compound the difficulty. Manual calculation to determine blend recipes often gives away costly quality in order to meet all specifications. An increasing number of gasoline blenders are profiting from the application of linear programming, which enables the producer to examine all possible combinations and quickly determine the most economical blend. Further, the standard use of LP to predict optimal blends contributes to the improvement of purchasing practices and inventory control.

LP MODEL FORMULATION

A linear programming model for gasoline blending is a mathematical representation, in the form of linear equations, or inequalities, of all known and estimated factors relevant to the production of the specified-quality gasoline. To demonstrate the method for formulating such a model, we postulate a specific problem: the production of two different gasolines — regular quality and premium quality — from a variety of materials each of which is available in limited supply.

INPUT DATA REQUIREMENTS

The following basic data is required to formulate the LP model:

- Blend specifications
- Inventory level of each raw material available
- Quality analysis of each raw material (at appropriate tetraethyl lead levels)
- Cost of each raw material
- Projected selling price of finished products

This input will provide a model for which maximum profit is computed. We might, instead of projected selling price, use an additional constraint — quantity of blend required. (For such a model, minimum cost is computed.)

The blend specifications for the final products are probably the most certain elements of the input data. Inventory levels, though fairly certain for the jobber, may be quite uncertain for the refiner operating continuously and with different grades of crude. (Specific yield requirements, if included in the model, may also be based on uncertainties.) In the sample problem, however, we shall assume specific inventory restrictions on each of the principal materials available for the blend.

Again, determining the cost of each component material is often a difficult problem when formulating a valid gasoline blending model. Fairly accurate prices may be determined from engineering projections, manufacturing costs, purchasing, or sales agreements which fix a market value for raw blend stocks. When accurate costing is impossible, estimates based on the comparative value of each fraction should be used so that at least a realistic ranking of material costs is incorporated.

SAMPLE PROBLEM — TWO-BLEND MODEL

We wish to produce two blends, regular and premium, from components which are available in specified amounts. Though in practice we can easily fix minimum or maximum amounts of blend to be produced, in this case the amounts are to be computed subject only to the limitations on component availability. Further, we shall establish tetraethyl lead (TEL) levels at 0.7 gm/gal. for regular and 2.0 gm/gal. for premium, and limit the alkylate and high severity reformate to the premium blend. Figure 1 lists the quantity of each material available together with its quality analysis and octane numbers at the established TEL levels. (Blending volatilities at 158°, 215°, and 240°F in Figure 1 were computed from the ASTM distillation of the components using the graphs of Naquin and Milwee, reference 2. Octane numbers at 0.7 and 2.0 gm/gal. in Figure 1 were obtained from tests at 0 and 3.0 gm/gal. and the DuPont TEL susceptibility chart A-21412, 8/61, which will be discussed in a subsequent section.)

The blend specifications and tolerances are listed in Figure 2. In the model matrix, we shall introduce the specification tolerances as safety factors. That is, we shall add the tolerance to a minimum specification and subtract it from a maximum

Component	Availability (bbl/day)	RVP	ASTM Blending Volatility* (%) @158°F	@215°F	@240°F	@300°F	Research Octane + TEL* @0.7 gm/gal.	@2 gm/gal.
Sour light virgin	13,830	6.2	7.5	38	77	96	79.5	80.5
Low severity reformate	4320	6.0	-5	19	35	56	90.5	93.6
High severity reformate	2410	6.2	-5	19	35	56	-	100.8
Light cat naphtha	16,400	4.5	11	50	74	88	95.5	97.9
Heavy cat naphtha	5250	0.0	-27	-20	-7	20	91.0	94.0
Alkylate	4000	4.5	10	27	97	100	-	103.3
Polymer	1900	8.0	7	25	62	87	99.5	100.5
Butane	As needed	72.0	130	101	100	100	101.5	104.0

*See text for sources of these values.

Figure 1. Availabilities and quality

Quality	Regular	Premium	Blending Tolerance
RVP maximum #	12.0	11.9	0.3
158°F volatility maximum %	33.0	29.0	3.0
215°F volatility minimum %	39.0	45.0	2.0
215°F volatility maximum %	51.0	56.0	2.0
240°F volatility minimum %	58.0	68.0	2.0
300 minimum %	78.0	88.0	2.0
Octane, minimum	88.5	97.5	0.5
TEL gm/gal.	0.7	2.0	---

Figure 2. Blend specifications and tolerances

specification. Thus, in the regular blend, the percentage volatility specification at 215°F, which is 39% minimum and 51% maximum with a tolerance of 2%, will be stated in the matrix as 41% minimum and 49% maximum.

We shall assume, for our sample problem, a basic distillate price of $4.00 per barrel and use the values listed in Figure 3 as the material costs in the model matrix. The information provided in Figures 1, 2, and 3 may be used to formulate a model matrix which can be solved for a maximum-profit blending of the available components into the two specified gasolines. The specifications, availability levels, TEL levels, and price data could, of course, be altered to reflect any new conditions.

A schematic of the LP model matrix is shown in Figure 4. The first row, which incorporates costs and anticipated selling prices, represents the objective function to be optimized and is designated COST (its exact meaning will be discussed below). The next several rows are used to establish inventory availability constraints. The rest of the rows

Component	Basis	Value ($/bbl)
Sour lt. virgin	Undercut distillate-flash correction	3.50
Low severity ref.	1/1.1 yield on undercut distillate + operating cost	4.30
High severity ref.	1/1.2 yield on undercut distillate + operating cost	4.55
Lt. cat naphtha	Undercut distillate-flash correction	3.50
Hvy. cat naphtha	Undercut distillate	4.00
Alkylate	Purchase price	7.00
Polymer	Fuel oil equivalent + shrinkage + operating cost	5.00
Butane	Fuel oil equivalent	2.00
Regular blend	Refinery realization	4.60
Premium blend	Refinery realization	5.45
TEL	Purchase price for 42 gm (42 gm/bbl. = 1 gm/gal.)	0.09

Note: Basic distillate price is $4.00 per barrel. The values used reflect yield, operating cost, and quality corrections as appropriate for the various components and are meant to represent fair raw-stock alternate disposition values.

Figure 3. Costs and prices

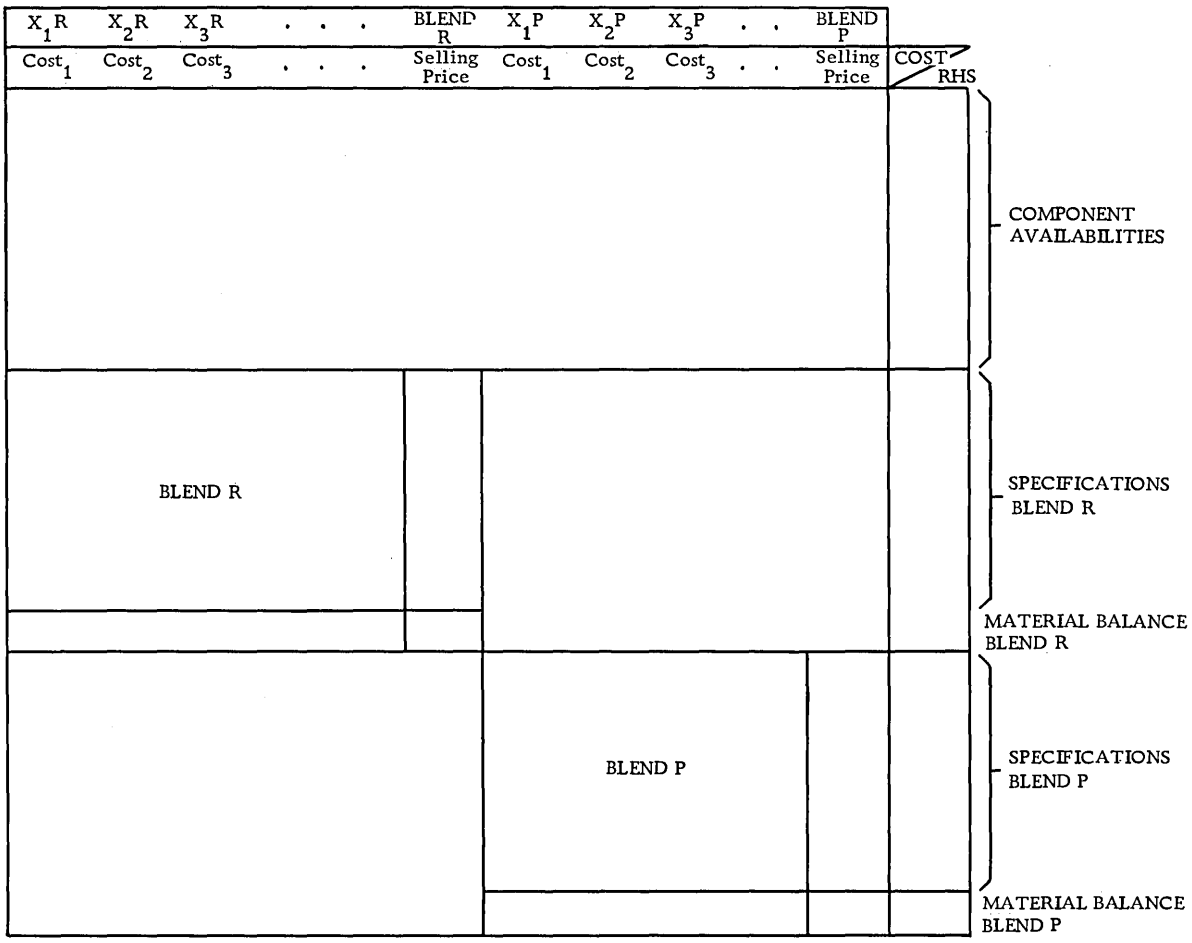

Figure 4. Schematic of a gasoline blending model matrix

provide the specification and material balance constraints for each of the blends to be produced. Since this schematic represents a two-product model, it includes two submatrices, each consisting of appropriate constraint rows to express the blend specifications for the corresponding gasoline. The raw materials are the same for both blends; hence, two activity columns are established for each material, with distinguishing mnemonics for each blend. For example, a material X_1 in Figure 4 is represented by a column designated X_1R for use in regular blend, and a column designated X_1P for use in premium blend.

It is convenient to use symbolic names (mnemonics) for the variables (material activity columns) and constraints (rows) which form the model. Hence, instead of identifying the variables as X_1, X_2, X_3, etc., we employ recognizable abbreviations. Sour light virgin used in the regular blend is designated RLVR. The same material used in premium blend is designated RLVP. A table of all the mnemonics that will be employed in the model matrix of our sample problem appears in Figure 5.

In Figure 5, the symbol in the row relation column indicates the type of constraint equation or inequality — that is, whether the sum of the terms to the left of the symbol in the row is equal to (=), less than or equal to ($\leq$), or greater than or equal to ($\geq$) the amount on the right-hand side. Minimize associated with the objective function (COST) row in the matrix indicates that the LP system will find the solution that minimizes this function while meeting all of the constraints.

Having established the input data (Figures 1, 2, and 3) and the mnemonics for the variables in our problem (Figure 4), we may now develop the constraint expressions. These fall into three groups:

- The cost constraint (objective function)
- Material availability constraints
- Blend specification constraints

VARIABLES (COLUMNS)

Name	Explanation
RLVR	Sour light virgin to regular, MB/D
LSRR	Low severity reformate to regular, MB/D
LCNR	Light cat naphtha to regular, MB/D
HCNR	Heavy cat naphtha to regular, MB/D
POLR	Polymer to regular, MB/D
BUTR	Butane to regular, MB/D
SPRR	On-specification regular blended, MB/D
RLVP	Sour light virgin to premium, MB/D
LSRP	Low severity reformate to premium, MB/D
HSRP	High severity reformate to premium, MB/D
LCNP	Light cat naphtha to premium, MB/D
HCNP	Heavy cat naphtha to premium, MB/D
ALKP	Alkylate to premium, MB/D
POLP	Polymer to premium, MB/D
BUTP	Butane to premium, MB/D
SPPP	On-specification premium blended, MB/D

CONSTRAINTS (ROWS)

Name	Explanation	Row Relation
COST	Objective function, $1000/day	MINIMIZE
RLVA	Sour light virgin availability, MB/D	$\leq$
LSRA	Low severity reformate availability, MB/D	$\leq$
HSRA	High severity reformate availability, MB/D	$\leq$
LCNA	Light cat naphtha availability, MB/D	$\leq$
HCNA	Heavy cat naphtha availability, MB/D	$\leq$
ALKA	Alkylate availability, MB/D	$\leq$
POLA	Polymer availability, MB/D	$\leq$
REGB	Regular blend material balance, MB/D	=
PREMB	Premium blend material balance, MB/D	=
RRVPX	RVP max. specification — regular	$\leq$
R158X	Percent off at 158°F max. specification — regular	$\leq$
R215N	Percent off at 215°F min. specification — regular	$\geq$
R215X	Percent off at 215°F max. specification — regular	$\leq$
R240N	Percent off at 240°F min. specification — regular	$\geq$
R300N	Percent off at 300°F min. specification — regular	$\geq$
RR.7N	Research octane (@0.7 gm TEL/gal.) min. specification — regular	$\geq$
PRVPX	RVP max. specification — premium	$\leq$
P158X	Percent off at 158°F max. specification — premium	$\leq$
P215N	Percent off at 215°F min. specification — premium	$\geq$
P215X	Percent off at 215°F max. specification — premium	$\leq$
P240N	Percent off at 240°F min. specification — premium	$\geq$
P300N	Percent off at 300°F min. specification — premium	$\geq$
PR2.N	Research octane (@2.0 gm TEL/gal.) min. specification — premium	$\geq$

Note: MB/D = Thousands of barrels per day.

Figure 5. Mnemonic tables

The following sections discuss each constraint in detail. (The complete matrix, incorporating all the constraints to be discussed, is shown in Figure 6.)

Cost Constraint (Objective Function)

As discussed earlier, the objective function in this problem expresses total profit from production of the two gasoline blends. This is to be maximized.

The objective function may be stated:

(SPRR x selling price per barrel regular) + (SPPP x selling price per barrel premium)

- (total cost of materials used) = MAXIMUM

where:

SPRR = number of barrels of regular blend produced

SPPP = number of barrels of premium blend produced (total cost of materials used)

= (RLVR x cost per barrel of sour lt. virgin.) + (LSRR x cost per barrel of low. sev. ref.) + . . . and so on for all the materials listed as variables in Figure 5, with each mnemonic representing the number of barrels used in the blends.

For reasons which are not detailed here, it is more efficient, in terms of computer time, to minimize the objective function than to maximize it. Taking advantage of this, we shall reverse the sign of each term in the above equation; to do this we merely enter each material cost per barrel as a positive value and the selling price per barrel of each blend as a negative value. The LP system will then solve for a minimum value of this equation which will be a negative number whose absolute value represents the total profit. (Since it would sound illogical to minimize a function called profit, we shall name the objective function COST; it need only be remembered that reducing cost to the lowest feasible negative value is equivalent to raising profit to the highest feasible positive value.)

The objective function, COST, will be the first row in the matrix (Figure 6). The terms representing material cost are formed quite simply: the cost per barrel for each material from (Figure 3) is merely entered as a coefficient in the COST row under the column for that material. Now, for each of the two blends (regular and premium), we establish a total-blend-produced column (SPRR and SPPP, respectively). In the COST row under these columns, we must enter a coefficient representing the anticipated selling price of each blend. As Figure 3 indicates, the basic selling price (refinery realization) of the regular blend is $4.60 per barrel. But we established that the TEL level for regular must be 0.7 gm/gal., and that it costs $0.09 per barrel to produce a TEL level of 1 gm/gal. Hence the net realization per barrel of regular blend produced will be

$4.60 - 0.7 ($0.09) = $4.537.

Similarly, the net realization per barrel of premium blend with a 2 gm/gal. TEL level is

$5.45 - 2 ($0.09) = $5.27.

Thus, the terms representing net realization from production are incorporated in the objective function by entering - 4.537 and - 5.27 as COST coefficients in the SPRR and SPPP columns, respectively.

The complete objective function shown in the COST row of the matrix can then be expressed as:

3.50 RLVR + 4.30 LSRR + 3.50 LCNR
+ 4.00 HCNR + 5.00 POLR + 2.00 BUTR
- 4.537 SPRR + 3.50 RLVP + 4.30 LSRP
+ 4.55 HSRP + 3.50 LCNP + 4.00 HCNP
+ 7.00 ALKP + 5.00 POLP + 2.00 BUTP
- 5.27 SPPP

= COST (MINIMIZE)

where the mnemonic variables represent the number of barrels of material used or blend produced (as defined earlier). The solution will minimize

	RLVR	LSRR	LCNR	HCNR	POLR	BUTR	SPRR	RLVP	LSRP	HSRP	LCNP	HCNP	ALKP	POLP	BUTP	SPPP		RHS
COST	3.5	4.3	3.5	4.0	5.0	2.0	-4.537	3.5	4.3	4.55	3.5	4.0	7.0	5.0	2.0	-5.27		MINIMIZE
RLVA	1							1									≤	13.83
LSRA		1							1								≤	4.32
HSRA										1							≤	2.41
LCNA			1								1						≤	16.40
HCNA				1								1					≤	5.25
ALKA													1				≤	4.00
POLA					1									1			≤	1.90
REGB	1	1	1	1	1	1	-1										=	0
PREMB								1	1	1	1	1	1	1	1	-1	=	0
RRVPX	6.2	6.0	4.5	0.0	8.0	72.0	-11.7										≤	0
R158X	7.5	-5.0	11.0	-27.0	7.0	130.0	-30.0										≤	0
R215N	38.0	19.0	50.0	-20.0	25.0	101.0	-41.0										≥	0
R215X	38.0	19.0	50.0	-20.0	25.0	101.0	-49.0										≤	0
R240N	77.0	35.0	74.0	-7.0	62.0	100.0	-60.0										≥	0
R300N	96.0	56.0	88.0	20.0	87.0	100.0	-80.0										≥	0
RR.7N	79.5	90.5	95.5	91.0	99.5	101.5	-89.0										≥	0
PRVPX								6.2	6.0	6.2	4.5	0.0	4.5	8.0	72.0	-11.6	≤	0
P158X								7.5	-5.0	-5.0	11.0	-27.0	10.0	7.0	130.0	-26.0	≤	0
P215N								38.0	19.0	19.0	50.0	-20.0	27.0	25.0	101.0	-47.0	≥	0
P215X								38.0	19.0	19.0	50.0	-20.0	27.0	25.0	101.0	-54.0	≤	0
P240N								77.0	35.0	35.0	74.0	-7.0	97.0	62.0	100.0	-70.0	≥	0
P300N								96.0	56.0	56.0	88.0	20.0	100.0	87.0	100.0	-90.0	≥	0
PR2.N								80.5	93.6	100.8	97.9	94.0	103.3	100.5	104.0	-98.0	≥	0

Figure 6. LP model matrix for two-blend problem

COST — that is, yield a negative COST value with the largest feasible absolute magnitude. With the sign changed from negative to positive, this COST value represents the maximum feasible profit.

Availability Constraints

We have formulated this example so that the total amount blended will depend on the availability and cost of each component. In practice, we might also establish upper or lower limits on the desired yield for each blend (as is done in the more complex example described in later sections of this manual). The formulation used for this simple example, however, will result in a solution providing the ratio between regular and premium blends (within the specification and availability constraints) which produces the largest profit.

The inventory availability constraints can be formulated in the matrix very simply. For example, the total amount of sour light virgin available is limited to 13,830 barrels per day (from Figure 1). This amount may be distributed between the regular blend and the premium blend. Thus, the total number of barrels of sour light virgin used in the regular blend (RLVR) and in the premium blend (RLVP) must not exceed 13.83 thousand barrels per day, or:

$$RLVR + RLVP \leq 13.83 \text{ MB/D.}$$

To express this limitation in the matrix, we establish a sour light virgin availability constraint row (RLVA) with 13.83 as its right-hand side and the coefficient 1 in the RLVR and RLVP columns.

Each of the remaining materials (except butane) is similarly bounded by summing the quantities used in regular and premium and setting the total equal to or less than the quantity available in MB/D (note that high severity reformate and alkylate are used only in the premium blend):

Low severity reformate availability (LSRA):	$LSRR + LSRP \leq 4.32$
High severity reformate availability (HSRA):	$HSRP \leq 2.41$
Light cat naphtha availability (LCNA):	$LCNR + LCNP \leq 16.40$
Heavy cat naphtha availability (HCNA):	$HCNR + HCNP \leq 5.25$
Alkylate availability (ALKA):	$ALKP \leq 4.00$
Polymer availability (POLA):	$POLR + POLP \leq 1.90$

It should be mentioned that the IBM 1620 LP system (and other IBM LP systems) actually provides for the bounding of column activities (variables) without the use of constraint rows. This feature reduces the effective size of a matrix, conserving computer time and storage capacity, and thus permits the processing of very large problems which would otherwise be impossible or uneconomical. (The use of bounded variables is illustrated in the three-blend sample problem to be discussed in a subsequent section.)

Having established the availability constraints for the components, we must now formulate a material balance expression for each blend which will constrain the quantity of final blend to the availability of components. We employ the total-blend-produced variables (SPRR and SPPP) which were established for the profit formulation. The sum of all components used in regular equals the total regular blended (SPRR), and the sum of all components used in premium equals the total premium blended (SPPP). Solving for a zero right-hand side, we obtain the following material balance equations:

$$RLVR + LSRR + LCNR + HCNR + POLR + BUTR - SPRR = 0$$

and

$$RLVP + LSRP + HSRP + LCNP + HCNP + ALKP + POLP + BUTP - SPPP = 0.$$

These appear in the matrix as rows REGB and PREMB, respectively.

The foregoing availability and material balance formulations, when combined with specifications formulations, insure a solution which will meet specifications within availability limits. We now need to consider the somewhat more complex specification formulations.

Specification Constraints

We shall assume for this sample problem that the various quality factors of each of the components blend linearly; that is, if one barrel of quality 10 is blended with one barrel of quality 20, the result will be two barrels of quality 15. The variations from such results observed in practice are treated later in the section devoted to model refinement techniques.

In order to ensure generally on-specification blends, we shall, as previously indicated, use the blend tolerances as safety factors. For instance, the RVP (Reid vapor pressure) specification for regular blend must not exceed 12 with a blend

tolerance of 0.3 (from Figure 2); therefore, we shall establish the maximum RVP for regular in the model matrix as 12 - 0.3, or 11.7. When a blend tolerance is associated with a minimum specification, as for the 215°F, 240°F, and 300°F volatility percentages, the blend tolerance is added to the minimum as a safety factor. Hence the minimums for 215°F, 240°F, and 300°F regular volatility percentages are 39 + 2, 58 + 2, and 78 + 2, or 41, 60, and 80, respectively.

Linear expressions serving to constrain the final blend to specifications can be formulated quite easily. For each quality factor (such as RVP number), the analysis values listed in Figure 1 may be multiplied by the quantities of the corresponding components in the blend, and the sum of these products expresses the total value of that quality factor in the final blend, which must meet the specification listed in Figure 2. Recall that there is a matrix column to represent the quantity of each raw material to be included in the blend recipe, and one to represent the total quantity of each blend. Using the column mnemonics as the variables, then, we may express the RVP maximum specification for regular (corrected for tolerance) as:

$$6.2\ \text{RLVR} + 6.0\ \text{LSRR} + 4.5\ \text{LCNR} + 0\ \text{HCNR} + 8.0\ \text{POLR} + 72.0\ \text{BUTR} - 11.7\ \text{SPRR} \leq 0$$

This specification constraint is then incorporated in the matrix by establishing a row (RRVPX) with the above coefficients in the appropriate columns and 0 as the right-hand side. Applying the same method to each of the blend specifications, we produce the necessary set of matrix rows. Thus, the maximum specification for volatility percentage at 158°F for regular is expressed as:

$$7.5\ \text{RLVR} - 5.0\ \text{LSRR} + 11.0\ \text{LCNR} - 27.0\ \text{HCNR} + 7.0\ \text{POLR} + 130.0\ \text{BUTR} - 30.0\ \text{SPRR} \leq 0,$$

which appears in the matrix as row R158X. For premium blend, the maximum 158°F volatility percentage specification is expressed as:

$$7.5\ \text{RLVP} - 5.0\ \text{LSRP} - 5.0\ \text{HSRP} + 11.0\ \text{LCNP} - 27.0\ \text{HCNP} + 10.0\ \text{ALKP} + 7.0\ \text{POLP} + 130.0\ \text{BUTP} - 26.0\ \text{SPPP} \leq 0,$$

which appears in the matrix as row P158X.

When all the blend specification and material availability constraints have been incorporated into the matrix, the problem formulation is complete.

Input Listing

Once the model matrix is formulated, the data are keypunched, and an input listing is prepared. The input listing for the formulated two-blend problem matrix appears in Figure 7.

The first section of the listing (ROW.ID) identifies the constraint rows and the type of row relation: "+" indicates an equal-to-or-less-than relation, "-" indicates an equal-to-or-greater-than relation, and "O" (blank) indicates an equality.

The second section (MATRIX) lists the coefficients, identified by column and row names. The right-hand sides for each constraint are listed in the third section (FIRST.B); where no entry appears values are assumed zero.

Summary of Sample Problem

In the foregoing discussion, we have demonstrated the application of linear programming techniques to gasoline blending by constructing an LP model designed to solve a typical production problem. In the following section we shall describe the output reports produced by the computer LP system, upon solution of our sample problem, and discuss the interpretation and analysis of these reports.

The sample problem was simplified by stating that only two blends are required, and by assuming linear octane blending and predetermined TEL levels. However, the LP model formulated here can be readily expanded to include several blends, octane weighting techniques, and TEL formulations as well. Indeed, the usefulness of the LP technique increases with the complexity of the problem. In a later section, we shall formulate and solve a larger and more complex problem as a further illustration of LP capabilities.

Construction of the basic LP model entails little more than organizing, in a special format, the data historically used in calculating gasoline blends. Once formulated and converted to input media for computer processing, the model becomes a master record. It can be updated regularly to account for new conditions such as the addition or deletion of activities, changes in inventory constraints, changes in costs, and changes in specifications.

OUTPUT REPORTS

The linear programming system may employ the input data to compute a variety of output reports. We are here principally concerned with four basic reports which the system produces:

- Basis variables report
- Slacks report
- DO.D/J report
- Cost range report

```
ROW.ID
          COST
        + RLVA
        + LSRA
        + HSRA
        + LCNA
        + HCNA
        + ALKA
        + POLA
          RBAL
          PBAL
        + RRVPX
        + R158X
        - R215N
        + R215X
        - R240N
        - R300N
        - RR.7N
        + PRVPX
        + P158X
        - P215N
        + P215X
        - P240N
        - P300N
        - PR.2N
MATRIX
      RLVR  COST     3.50
      RLVR  RLVA     1.0
      RLVR  RBAL     1.0
      RLVR  RRVPX    6.2
      RLVR  R158X    7.5
      RLVR  R215N    38
      RLVR  R215X    38
      RLVR  R240N    77
      RLVR  R300N    96
      RLVR  RR.7N    79.5
      LSRR  COST      4.30
      LSRR  LSRA      1.
      LSRR  RBAL     1.0
      LSRR  RRVPX    6.0
      LSRR  R158X    -5.
      LSRR  R215N    19.
      LSRR  R215X    19.
      LSRR  R240N    35.
      LSRR  R300N    56.
      LSRR  RR.7N    90.5
      LCNR  COST     3.50
      LCNR  RBAL     1.0
      LCNR  LCNA     1.
      LCNR  RRVPX    4.5
      LCNR  R158X    11.
      LCNR  R215N    50.
      LCNR  R215X    50.
      LCNR  R240N    74.
      LCNR  R300N    88.
      LCNR  RR.7N    95.5
      HCNR  COST     4.00
      HCNR  HCNA     1.0
      HCNR  RBAL     1.0
      HCNR  RRVPX    0.
      HCNR  R158X-   27.
      HCNR  R215N-   20.
      HCNR  R215X-   20.
      HCNR  R240N-    7.
      HCNR  R300N    20.
      HCNR  RR.7N    91.0
      POLR  COST     5.00
      POLR  POLA     1.0
      POLR  RBAL     1.0
      POLR  RRVPX    8.
      POLR  R158X    7.
      POLR  R215N    25.
      POLR  R215X    25.
      POLR  R240N    62.
      POLR  R300N    87.
      POLR  RR.7N    99.5
      BUTR  COST     2.00
      BUTR  RRVPX    72.
      BUTR  RBAL     1.0
      BUTR  R158X    130.
      BUTR  R215N    101.
      BUTR  R215X    101.
      BUTR  R240N    100.
      BUTR  R300N    100.
      BUTR  RR.7N    101.5
      SPRR  COST     -4.537
      SPRR  RBAL     -1.
      SPRR  RRVPX    -11.7
      SPRR  R158X    -30.
      SPRR  R215N    -41.
      SPRR  R215X    -49.
      SPRR  R240N    -60.
      SPRR  R300N    -80.
      SPRR  RR.7N    -89.
      RLVP  COST     3.50
      RLVP  RLVA     1.0
      RLVP  PBAL     1.0
      RLVP  PRVPX    6.2
      RLVP  P158X    7.5
      RLVP  P215N    38
      RLVP  P215X    38
      RLVP  P240N    77
      RLVP  P300N    96.0
      RLVP  PR.2N    80.5
      LSRP  COST     4.30
      LSRP  LSRA      1.
      LSRP  PBAL     1.0
      LSRP  PRVPX    6.0
      LSRP  P158X    -5.
      LSRP  P215N    19.
      LSRP  P215X    19.
      LSRP  P240N    35.
      LSRP  P300N    56.
      LSRP  PR.2N    93.6
      HSRP  COST     4.55
      HSRP  HSRA      1.
      HSRP  PBAL     1.0
      HSRP  PRVPX    6.2
      HSRP  P158X-   5.
      HSRP  P215N    19.
      HSRP  P215X    19.
      HSRP  P240N    35.
      HSRP  P300N    56.
      HSRP  PR.2N    100.8
      LCNP  COST     3.50
      LCNP  LCNA     1.0
      LCNP  PBAL     1.0
      LCNP  PRVPX    4.5
      LCNP  P158X    11.
      LCNP  P215N    50.
      LCNP  P215X    50.
      LCNP  P240N    74.
      LCNP  P300N    88.
      LCNP  PR.2N    97.9
      HCNP  COST     4.00
      HCNP  HCNA     1.0
      HCNP  PBAL     1.0
      HCNP  PRVPX    0.
      HCNP  P158X-   27.
      HCNP  P215N-   20.
      HCNP  P215X-   20.
      HCNP  P240N-    7.
      HCNP  P300N    20.
      HCNP  PR.2N    94.0
      ALKP  COST     7.00
      ALKP  ALKA     1.0
      ALKP  PBAL     1.0
      ALKP  PRVPX    4.5
      ALKP  P158X    10.
      ALKP  P215N    27.
      ALKP  P215X    27.
      ALKP  P240N    97.
      ALKP  P300N    100.
      ALKP  PR.2N    103.3
      POLP  COST     5.00
      POLP  POLA     1.0
      POLP  PBAL     1.0
      POLP  PRVPX    8.
      POLP  P158X    7.
      POLP  P215N    25.
      POLP  P215X    25.
      POLP  P240N    62.
      POLP  P300N    87.
      POLP  PR.2N    100.5
      BUTP  COST     2.00
      BUTP  PBAL     1.0
      BUTP  PRVPX    72.
      BUTP  P158X    130.
      BUTP  P215N    101.
      BUTP  P215X    101.
      BUTP  P240N    100.
      BUTP  P300N    100.
      BUTP  PR.2N    104.0
      SPPP  COST     -5.27
      SPPP  PBAL     -1.
      SPPP  PRVPX    -11.6
      SPPP  P158X    -26.
      SPPP  P215N    -47.
      SPPP  P215X    -54.
      SPPP  P240N    -70.
      SPPP  P300N    -90.
      SPPP  PR.2N    -98.
FIRST.B
                     RLVA    13.83
                     LSRA     4.32
                     HSRA     2.41
                     LCNA   16.40
                     HCNA   5.25
                     ALKA     4.00
                     POLA     1.90
EOF
MIN
OUTPUT
CHECK.
DO.D/J
COST.R
```

Figure 7. Input listing

Each of these reports is discussed and illustrated below.

BASIS VARIABLES AND SLACKS REPORTS

The basis variables report (Figure 8) provides a list of all the activities in the "basis" (that is, the set of all materials appearing at a nonzero level in the optimal blends) and indicates the quantity of each material used for each of the blends. The slacks report (Figure 9) provides a list of all the row mnemonics -- that is, all the inventory availability and specification constraints -- and indicates how much of each available material was not used and the amount by which each quality specification was exceeded. If the maximum quantity of a material was used, or if a specification was met at a bound, the slacks report provides a figure called the simplex multiplier, which is significant in the DO.D/J report discussed later.

The letters in the TYPE column of these reports indicate the condition of the variables. The letter "F" indicates solution at an intermediate level, "W" indicates solution at a lower bound, and "G" indicates solution at an upper bound. The signs have been taken from the input listing.

The standard solution printout given in Figures 8 and 9 can be more readily interpreted when reorganized. One such reorganization is shown in Figure 10. In this figure, values in the regular and premium "Component Disposition" columns, representing amounts used, have been taken from the ACTIVITY LEVEL column of the basis variables report (Figure 8); the "Unused" component values are taken from the ACTIVITY LEVEL column of the slacks report (Figure 9); and the "Marginal Value" items are from the SIMPLEX MULT. column of Figure 9.

The component marginal values given in Figure 10 may be interpreted as the premium that could be paid for additional stocks. For example, since the marginal value of light cat naphtha is $2.16 per bbl., and its cost in the model was set at $3.50 per bbl., the refinery could pay up to $5.66 per bbl. for additional light cat naphtha over and above its present availability of 16.4 MB/D. Such a high marginal value indicates that a new running plan which makes more light cat naphtha available for gasoline blending would undoubtedly be profitable.

VARBLS	TYPE	NAME	ACTIVITY LEVEL
	F	RLVR	13.164
	F	LSRR	4.320
	F	LCNR	10.789
	F	HCNR	2.764
	F	BUTR	3.434
	F	SPRR	34.471
	F	RLVP	.666
	F	LCNP	5.611
	F	ALKP	.412
	F	POLP	1.900
	F	BUTP	.881
	F	SPPP	9.469

Figure 8. Basis variables report

SLACKS	TYPE	NAME	ACTIVITY LEVEL	SIMPLEX MULT.
	F	COST	49.849-	
	+W	RLVA		.907
	+W	LSRA		.198
	+F	HSRA	2.410	
	+W	LCNA		2.156
	+F	HCNA	2.486	
	+F	ALKA	3.588	
	+W	POLA		.574
	G	RBAL	.000	.107
	G	PBAL	.000	9.464
	+W	RRVPX		.078
	+F	R158X	466.560	
	-W	R215N		.025-
	+F	R215X	275.764	
	-F	R240N	219.012	
	-F	R300N	96.107	
	-W	RR.7N		.051-
	+W	PRVPX		.075
	+F	P158X	47.575	
	-F	P215N	8.362	
	+F	P215X	57.923	
	-F	P240N	49.445	
	-W	P300N		.069-
	-W	PR.2N		.096-

Figure 9. Slacks report

The basis variables and slacks reports also provide important data which enables the producer to determine the cost of quality and the amount of quality giveaway. The chart of Figure 11 tabulates the cost of specified quality (taken from the SIMPLEX MULT. column of the slacks report), the slack in quality-MB/D (which has little physical interpretation and which was taken from the ACTIVITY LEVEL column of the slacks report) and the true quality of the blend. This last is determined as follows: the slack is divided by the total amount of the grade blended, and the result is either subtracted from or added to the tolerance-adjusted specification, depending on whether the specification is a maximum or a minimum, respectively. For example, the 158°F maximum volatility specification for regular of 30% was exceeded by 467 quality-MB/D. Since 34.471 MB/D

Component	Component Disposition			Marginal Value ($/bbl)
	Regular (MB/D)	Premium (MB/D)	Unused (MB/D)	
Sour light virgin	13.164	0.666	---	0.91
Low severity reformate	4.320	---	---	0.20
High severity reformate	---	---	2.410	---
Light cat naphtha	10.789	5.611	---	2.16
Heavy cat naphtha	2.764	---	2.486	---
Alkylate	---	0.412	3.588	---
Polymer	---	1.900	---	0.57
Butane	3.434	0.881	---	---
TOTAL	34.471	9.469	---	---

Figure 10. Component disposition

	Regular			Premium		
	Cost of Specs. ($/Q-bbl)	Slack (Q-MB/D)	Qual. (Q)	Cost of Specs. ($/Q-bbl)	Slack (Q-MB/D)	Qual. (Q)
RVP	0.078		11.7	0.075		11.6
158 MAX		467	16.5		48	20.9
215 MIN	-0.025		41.0		8	47.8
240 MIN		219	66.4		49	75.2
300 MIN		96	82.8	-0.069		90.0
Octane	-0.051		89.0	-0.096		98.0

Figure 11. Quality

of regular was blended, the true quality of the blend for 158° F maximum volatility is

$$30 - 467/34.471 = 16.5.$$

The costs of the octane specifications are the most interesting figures in this table. The cost of octane from components is 5.1¢ per Q-bbl. for regular and 9.6¢ per Q-bbl. for premium, at TEL levels of 0.7 and 2.0 gm/gal., respectively. We should calculate the cost of octane from TEL in order to determine whether the TEL levels chosen for the model contribute to a maximum profit blend. If they were well chosen, there will be little difference in the cost of octane from components and the cost of octane from TEL. If the TEL levels are too low, then the cost of octane from TEL will be substantially less than the cost of octane from components, and, of course, the reverse will be true if the TEL levels are too high.

To evaluate the cost of octane from TEL in our example, we determine the octane number of each of the blends at 0 and 3 gm/gal. TEL by multiplying

the amount of each component used in each blend by its octane at 0 and 3 gm/gal. TEL, and dividing by the total amount of each blend produced. In short, we calculate, by averaging each of these component octanes, the octane of the two specified blends at 0 and 3 gm/gal. TEL levels. These values, plotted on the DuPont TEL susceptibility chart and joined by lines, provide the octane-versus-TEL relationships over the range from 0 to 3 gm/gal. (Figure 12).

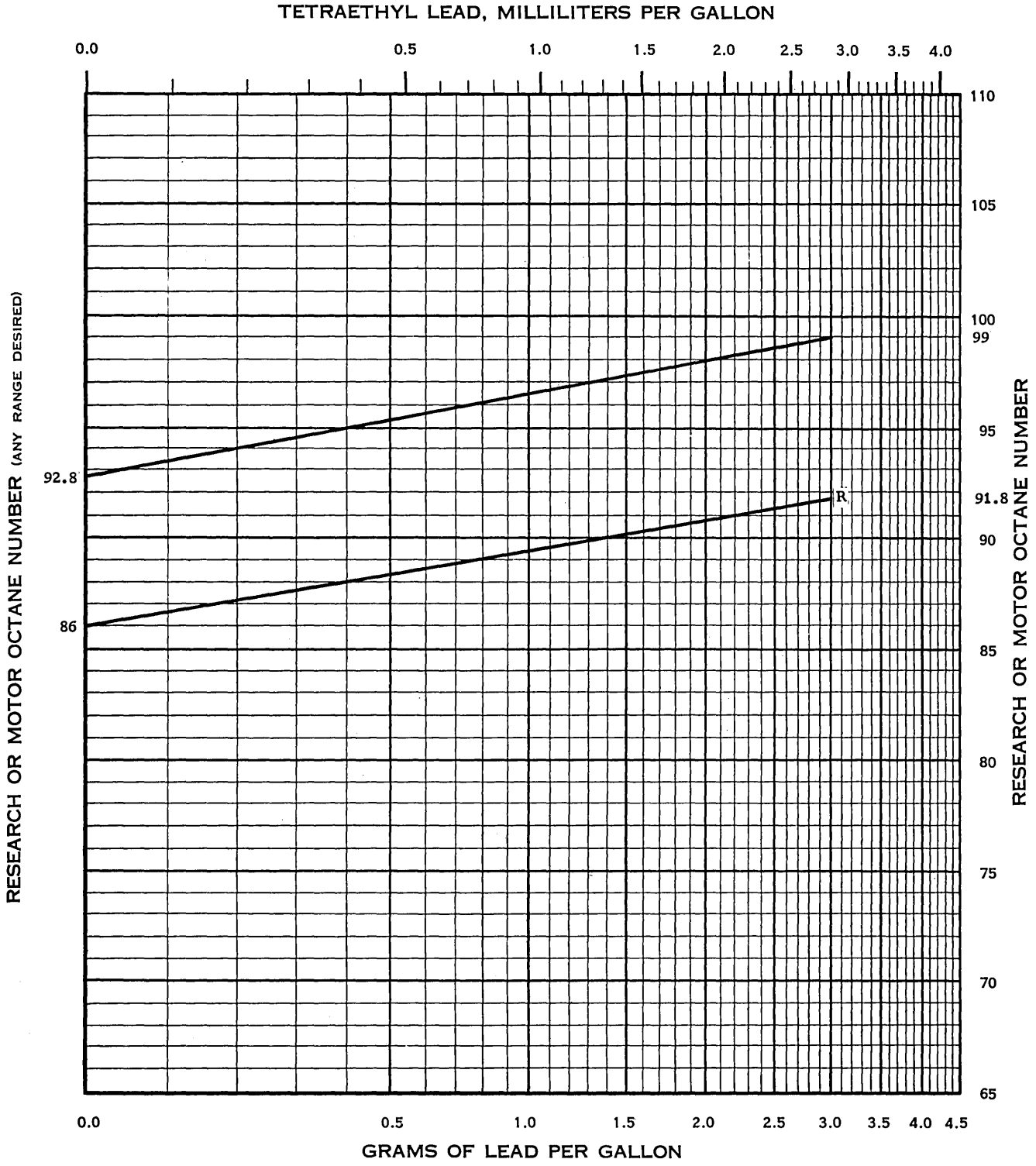

Figure 12. TEL motor antiknock susceptibility chart.
Form, courtesy of E. I. du Pont de Nemours & Co. (Inc.)

The lines on the TEL susceptibility chart are replotted in linear rectangular coordinates in Figure 13. If we take the partial derivative of octane with respect to TEL at the specific TEL concentrations used in each blend, we determine the slope of the octane-versus-TEL curves at those levels (indicating the rate of octane appreciation per gm/gal. TEL). The cost of octane from TEL, then, can be determined easily as follows: We divide the cost of TEL per barrel at 1 gm TEL/gal. ($0.09), by the rate of octane appreciation at the given TEL levels.

The slope of the TEL response curve for regular at 0.7 gm/gal. TEL ($\Delta R/\Delta T$) = 2.4. The slope of the TEL respense curve for premium at 2.0 gm/gal. TEL = 1.2. Hence, the cost of octane from lead for each blend may be calculated:

$$\text{Cost}_R = \frac{0.09}{\text{Slope}_R} = \frac{0.09}{2.4} = 0.037$$

$$\text{Cost}_P = \frac{0.09}{\text{Slope}_P} = \frac{0.09}{1.2} = 0.075$$

where

Cost_R = cost of octane from lead (regular), $ per octane-bbl.

Cost_P = cost of octane by lead (premium), $ per octane-bbl.

Slope_R = slope of octane-versus-TEL graph at specified lead level (regular).

Slope_P = slope of octane-versus-TEL graph at specified lead level (premium).

The linear programming result gave octane costs from component blending as $0.051 and $0.096 per octane-bbl. for regular and premium at 0.7 and 2 gm/gal. levels, respectively. Calculation reveals that, at the 0.7 gm/gal. TEL level for the regular

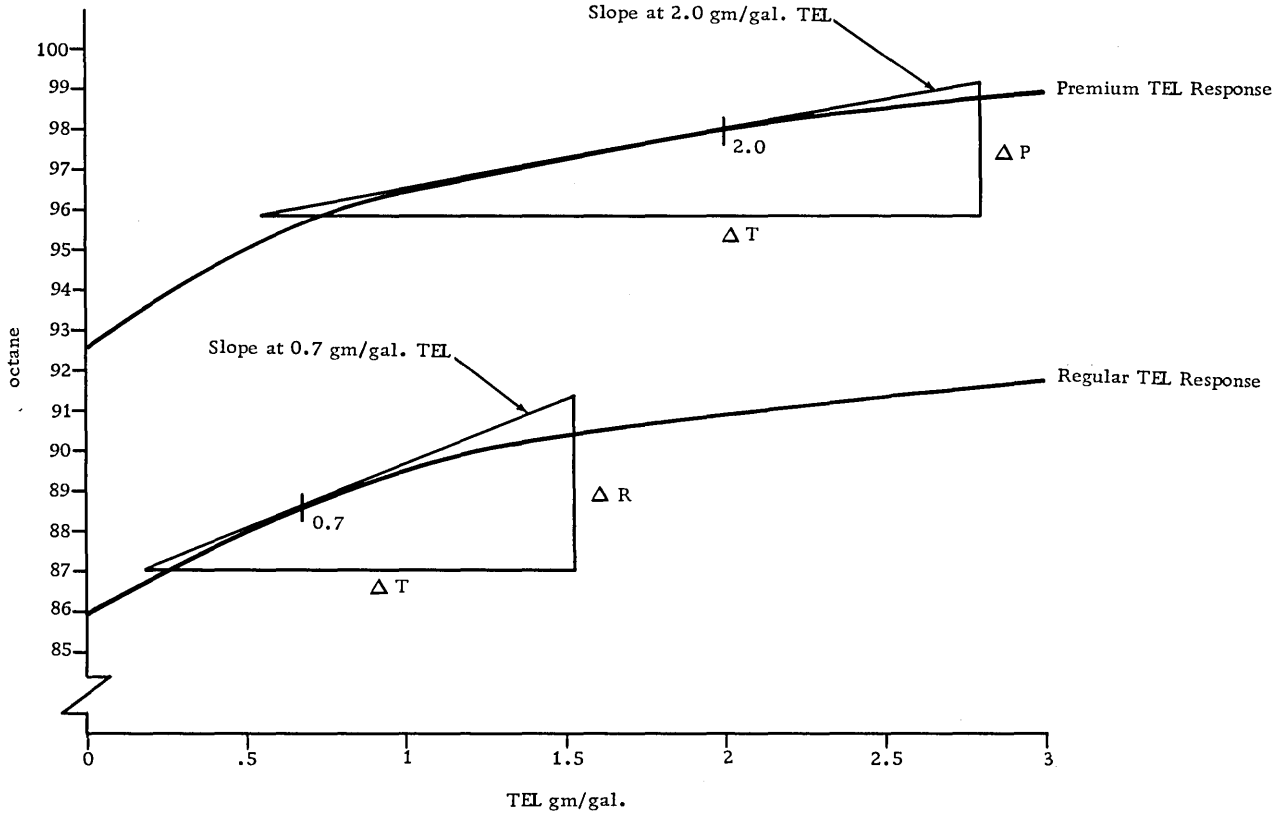

Figure 13. TEL response curves

blend, octane appreciation from lead costs $0.037, while at 2 gm/gal. TEL (premium), octane appreciation from lead costs 0.075 per octane-bbl. Clearly, higher TEL levels for both regular and premium would result in specified octane numbers at less cost. The amount of upward adjustment remains a matter of judgment since new component qualities will result in alternative blend compositions. Desirable changes in component availability can be introduced into the model matrix at the same time.

RE-SOLUTION

The data provided by the basis variables and slacks reports and the TEL calculations suggests a re-solution with the following changes in the model matrix:

Component Availabilities

$$LCNA \leq 18.40 \text{ MB/D}$$

$$RLVA \leq 14.83 \text{ MB/D}$$

Tetraethyl lead levels were established at 0.9 gm/gal. for regular and 2.5 gm/gal. for premium. The component octane numbers at these levels are given in Figure 14.

The re-solution can be accomplished quite rapidly by punching new cards only for the altered values and introducing the new values into the computer-stored problem. The altered input listing appears in Figure 15. The new COST

Components	Octane Quality	
	0.9 gm/gal.	2.5 gm/gal.
RLV	79.8	80.8
LSR	91	94.4
HSR	--	101.5
LCN	95.9	98.5
HCN	91.8	94.7
ALK	--	104.3
POL	99.7	100.8
BUT	102	104.6

Figure 14. Octane numbers at 0.9 gm/gal. and 2.5 gm/gal. TEL levels

```
REVISE
MATRIX
       RLVR   RR.7N    79.8
       LSRR   RR.7N    91.
       LCNR   RR.7N    95.9
       HCNR   RR.7N    91.8
       POLR   RR.7N     99.7
       BUTR   RR.7N   102.
       SPRR   COST      -4.519
       RLVP   PR.2N    80.8
       LSRP   PR.2N    94.4
       HSRP   PR.2N   101.5
       LCNP   PR.2N    98.5
       HCNP   PR.2N    94.7
       ALKP   PR.2N   104.3
       POLP   PR.2N   100.8
       BUTP   PR.2N   104.6
       SPPP   COST      -5.225

FIRST.B
              RLVA     14.83
              LCNA      18.40
EOF
MIN
OUTPUT
CHECK.
DO.D/J
COST.R
```

Figure 15. Revisions for re-solution of the model matrix

figures for regular (SPRR) and premium (SPPP) reflect the increased cost due to additional TEL. Re-solution with new availabilities and new TEL levels produces the output tabulation of Figure 16.

The new overall profit is $55,255 per day (substantially higher than the $49,849 profit of the first solution). If we multiply the quantity of each component used in regular by its cost, sum the costs, divide by the quantity of regular produced, and subtract the result from the selling price of regular, we determine the profit per barrel — which, in this case, is $1.03. Premium was blended at a profit of $1.57 per barrel. Further, in the re-solution, the cost of octane from lead is close to the cost of octane from component allocation. The high marginal value associated with light cat naphtha indicates that a second re-solution with even more of that component available will produce a more profitable blend. Additional information is produced by the LP output in the DO.D/J and cost range reports discussed below.

Component	Component Disposition			Marginal Value ($/bbl)
	Regular (MB/D)	Premium (MB/D)	Unused (MB/D)	
Sour light virgin	13.662	1.168		0.904
Low severity reformate	4.320	---		0.199
High severity reformate	---	---	2.410	
Light cat naphtha	9.760	8.640		2.170
Heavy cat naphtha	2.452	---	2.798	
Alkylate	---	0.326	3.674	
Polymer	---	1.900		0.560
Butane	3.296	1.272	---	---
TOTAL	33.490	13.305		

	Qualities					
	Regular			Premium		
	Cost of Specs. ($/Q-bbl)	Slack (Q-MB/D)	Qual. (Q)	Cost of Specs. ($/Q-bbl)	Slack (Q-MB/D)	Qual. (Q)
RVP MAX	0.079		11.7	0.074		11.6
158 MAX		454	16.4		60	21.5
215 MIN	-0.026		41		36	49.7
240 MIN		228	66.8		74	75.6
300 MIN		111	83.3	-0.065		90
Octane	-0.051		89	-0.094		98

Figure 16. Component disposition and quality from re-solution

DO.D/J REPORT

The DO.D/J report (Figure 17) consists of two parts. The first part (VBLS) lists all the column activities — raw materials in this case — which are solved at a bound. When the material is at a lower bound, ordinarily zero, the report indicates for each material its current cost and the amount this cost must drop, as well as the actual cost to which it must drop, before the material may be introduced into the basis. When an upper bound restrains the raw material, the report indicates the highest price at which that material would remain in the basis at its upper bound.

```
DO.D/J
 VBLS    TYPE  NAME   CURRENT COST   REDUCED COST    BASIS VALUE
            W  POLR          5.000           .619          4.381
            W  LSRP          4.300          1.419          2.881
            W  HSRP          4.550           .818          3.732
            W  HCNP          4.000          2.801          1.199
 ROWS    TYPE  NAME   INCR B VALUE   DECR B VALUE
            +  RLVA           .904
            +  LSRA           .199
            +  HSRA
            +  LCNA          2.170
            +  HCNA
            +  ALKA
            +  POLA           .560
               RBAL           .166
               PBAL          9.006
            +  RRVPX          .079
            +  R158X
            -  R215N                          .026
            +  R215X
            -  R240N
            -  R300N
            -  RR.7N                          .051
            +  PRVPX          .074
            +  P158X
            -  P215N
            +  P215X
            -  P240N                          .065
            -  P300N                          .094
            -  PR.2N
```

Figure 17. DO.D/J report from re-solution

Referring to Figure 17, we see, for example, that no polymer (POLR) is used in the regular blend, and that polymer will not enter the regular blend unless its cost falls below $4.381 per barrel. Similarly, no low severity reformate (LSRP) is used in the premium blend, and none will be used unless its cost falls below $2.881 per barrel. High severity reformate (HSRP) will not enter the blend unless its price falls below $3.732 per barrel.

The second part of the DO.D/J report (ROWS) provides a list of all the row (right-hand-side) mnemonics, and for each equation (and each inequality solved at a bound) indicates the "cost" of changing the right-hand side by one unit. (This "cost" is the value of the simplex multiplier in the slacks report, Figure 9.) If, for example, additional sour light virgin (RLVA) were available, the total profit would increase by 90.4¢ per barrel of sour light virgin added in the neighborhood of the optimal solution.

This report also reveals the price of specified quality. In effect, it "costs" 7.9¢ per unit of RVP per barrel to meet the RVP specification for regular (RRVPX). Re-solution with a relaxed specification would doubtless produce a more profitable blend. Such indications alert the producer to those specifications which are most costly and provide good indications of where permissible quality changes should be made.

Thus, the DO.D/J report establishes the price of quality, by indicating the cost of meeting a specification at a bound. Where these costs are high, re-solution with a slight relaxation of specification may produce considerable savings. In actual practice, the producer may also find it desirable to force the allocation of certain materials in order to exercise proper inventory control. The DO.D/J report indicates the penalties which would result from such forcing of components into a blend, rendering it nonoptimal. However, since these

costs hold only in the neighborhood of the optimal solution, the producer should re-solve, using a revised matrix input.

COST RANGE REPORT

The cost range (COST. R) report (Figure 18) indicates for each component that is included in the basis (optimal blends) the following data: current cost, highest cost before its quantity in the optimal solution changes, what other component would enter the solution at that highest cost, lowest cost before its quantity in the optimal solution changes, what other component would enter the solution at that lowest cost.

The quantity of each component in the optimal solution (given by the basis variables report) will remain unchanged within the cost range indicated by the cost range report. For example, 13.662 MB/D of sour light virgin (RLVR) would be allocated to regular even if it cost $4.141 per barrel instead of $3.50. Were sour light virgin to exceed $4.141 per barrel, the allocation ratio between regular and premium would change through a complex shifting of blend components which would result in high severity reformate (HSRP) entering the optimal premium blend. Were its price to drop below $1.658 per barrel, a slack (PR. 2N) would appear in the premium octane specification. This indicates that so much of the sour light virgin would be used in the regular blend, that, given the component availabilities and prices of this model, the most economical blending would allow octane giveaway in the premium. (In practice, however, a re-solution might reveal that the most economical blending would produce no premium at all.) Note that the cost range figures for sour light virgin allocated to premium (RLVP) indicate that the reverse occurs. That is, above the price of $5.342, premium octane giveaway is economical, while below the price of $2.859, much of the component will be employed in the regular blend and high severity reformate (HSRP) will enter the premium blend.

Particularly in multiblend models of this kind, actual price changes which exceed the cost range indications justify a re-solution of the problem, since the complexity of allocation shifts within and between the blends are not revealed in the cost range report. The cost range report, however, does give a good indication of when such re-solutions are desirable.

OUTPUT REPORT SUMMARY

The various output reports furnished by the LP system thus not only provide a detailed listing of the specific optimal solution but also alert the producer to a variety of relationships, any one of which may profoundly influence the total profit from the blends. The computer enables the producer to re-solve the problem rapidly with a number of variations suggested by the output reports. He can, in effect, use the LP model as an aid in the solution of a series of different problems. What if the price of each of the components varies? What if certain inventory purchases are possible at specific prices? What if quality controls vary? The LP solutions provide information which enables the producer to make the most judicious policy

COST.R

COST.R NAME	CURRENT COST	HIGHEST COST	HI-VAR	LO-VAR	LOWEST COST
RLVR	3.500	4.141	HSRP	PR.2N	1.658
LSRR	4.300	4.499	LSRA		INFINITY-
LCNR	3.500	4.200	POLR	RR.7N	3.014
HCNR	4.000	5.164	POLR	LSRA	3.480
BUTR	2.000	4.670	HSRP	RR.7N	.538-
SPRR	4.519-	4.240-	HSRP	RR.7N	4.821-
RLVP	3.500	5.342	PR.2N	HSRP	2.859
LCNP	3.500	3.986	RR.7N	POLR	2.800
ALKP	7.000	13.978	POLA	HSRP	6.636
POLP	5.000	5.560	POLA		INFINITY-
BUTP	2.000	5.809	RR.7N	HSRP	.593
SPPP	5.225-	4.878-	POLA	HSRP	5.360-

Figure 18. Cost range report from re-solution

decisions in matters of refinery operation, purchasing, quality control, inventory control, and product research. LP techniques make possible continuous management study — resulting in decreased costs, increased efficiency, and maximum profits.

MODEL REFINEMENT TECHNIQUES

A number of refinements which contribute to more efficient and profitable operation can be introduced into the LP model for gasoline blending. These refinements, which generally increase accuracy, may be used to introduce methods for:

1. Linear approximations of TEL susceptibility
2. Linear formulation of composite quality specifications
3. Linear formulation of process yield components

We shall employ a number of these refinements in the construction of a large illustrative model matrix.

LINEAR APPROXIMATIONS FOR TEL SUSCEPTIBILITY

Tetraethyl lead does not linearly affect the octane of gasoline components. To evaluate with reasonable accuracy the effects on octane of a number of different TEL levels, we must empirically determine the octane of the component without TEL and with TEL at a fairly high level, say 3 gm/gal. The empirically determined octanes provide the basis for interpolation (permitting a close estimate of response over a range of TEL levels). This is accomplished by plotting the empirically determined values on standard TEL graph paper (see Figure 12); a straight line interconnecting the two values on this paper then indicates the approximate octane of the component at practical TEL levels.

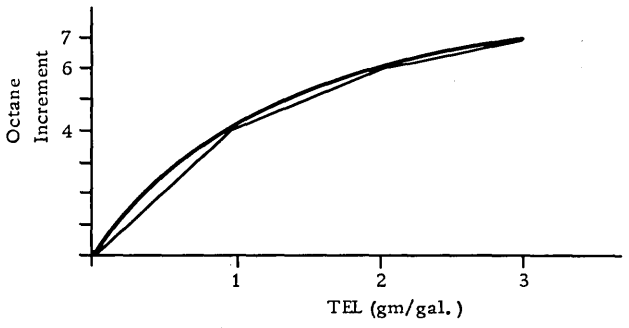

Figure 19. Estimated TEL curve for blend

In the sample problem, for which the model of Figure 6 was developed, the nonlinear effect of TEL on octane was handled by successive approximations. First the components were blended at specified TEL levels for regular and premium, and then the output reports were analyzed in order to compare the cost of octane from components to the cost of octane from lead. The analysis suggested that higher TEL levels would produce the specified octane at less cost, and the problem was re-solved for higher specific TEL levels. Again, analysis of the output reports is required to determine the relationship between the cost of octane from components and the cost of octane from lead. It is possible, however, to formulate linear expressions which approximate the TEL susceptibility curves for the blend, and include them in the model matrix, which then can be solved for the TEL level required in each of the specified blends. Three methods of different sensitivities will be discussed.

The first, simplest, and least sensitive method is based on an estimated TEL susceptibility curve for the finished blend. For example, assume the blend TEL response curve shown in Figure 19.

The octane number appreciates at approximately the rate of 4 numbers per gm/gal. between 0 and 1 gm/gal. TEL, at approximately 2 numbers per gm/gal. between 1 and 2 gm/gal. TEL, and at approximately 1 number per gm/gal. between 2 and 3 gm/gal. TEL. (Choosing closer intervals would result in more precise octane-versus-TEL relationships; however, since this method is based on an estimated TEL curve to begin with, it does not warrant closer intervals.) We can formulate three new activity columns L_1, L_2, L_3 which represent the amounts of tetraethyl lead added (in gm/gal.) between 0 and 1 gm/gal., between 1 and 2 gm/gal., and between 2 and 3 gm/gal. Each of these new variables has the same price, $0.09 per bbl. for 1 gm/gal. (or, in other words, the price of 42 grams).

Note: The reader should be on guard against possible misunderstanding as a result of the various units used in discussing TEL. Gram-per-gallon (gm/gal.) units are used on charts and in discussions of TEL. In the matrix formulations, however, TEL is measured in per-barrel "units" of 42 grams. Since there are 42 gal./bbl, concentrations in either case are numerically the same (for example, 1 gm/gal. = 1 unit/bbl) though the actual amounts and prices of TEL involved are different.

Octane of each component, then, is introduced into the octane specification row at a zero TEL level, but added to the octane specification row are three new terms:

$$4L_1 + 2L_2 + L_3$$

signifying that the octane number is increased at the rate of 4 per gm/gal. for any TEL added between 0 and 1 gm/gal., at the rate of 2 per gm/gal. for any TEL added between 1 and 2 gm/gal., and at the rate of 1 per gm/gal. for any TEL added between 2 and 3 gm/gal. Since the TEL price is the same for L_1, L_2, and L_3, all the L_1 (which adds most octane per dollar) will be used before any L_2, and similarly all L_2 will be used before any L_3. But, in order to establish proper limits on the TEL activities in the matrix, we must include constraints in the matrix which limit L_1, L_2, and L_3 to 1 unit/bbl. (42 gm/bbl. or 1 gm/gal.) each. That is, if SPRR turns out to be 10,000 bbl. (10 MB/D), then L_1, L_2 and L_3 are each limited to 10,000 units or 420,000 gms. for the entire blend. Thus:

$$L_1 - SPRR \leq 0$$

$$L_2 - SPRR \leq 0$$

$$L_3 - SPRR \leq 0$$

If this method were employed for TEL computation, then the octane specification for the regular blend in the sample model matrix (Figure 6) would appear as shown in Figure 20. This formulation would give the optimum TEL level for the desired blend if the estimated blend TEL response curve and the linearization of it were reasonably accurate.

A second method for formulating a linear approximation of the TEL response curve (developed by Healy — see reference 3) makes use of a related but slightly different technique. In this method, the quality for each component is introduced into the matrix at two distinct levels — say 1 and 3 gm/gal. Thus, two rows are required to establish the octane specification, one summing the octane of the components at 1 gm/gal. levels and one summing the octane of the components at 3 gm/gal. levels. Figure 21 shows the matrix formulation of the TEL approximation and Figure 22 illustrates its derivation.

In Figure 22 a linear approximation of TEL response at the two levels is established by determining the slope of a typical TEL response curve for the blend at the two TEL levels. The slope at 1 gm/gal. is 4, and at 3 gm/gal. the slope is 1. The model matrix formulation will result in a TEL approximation lying along the cross-hatched portion of the slope lines. It can be shown that the variation in slope for most TEL curves is sufficiently small so that these values can be used as reasonable approximations to cover the TEL response curves, averaged, of all the components. We can then establish the formulations, consisting of two rows, as follows:

$$79.5\ RLVR + 90.5\ LSRR + 95.5\ LCNR + 93\ HCNR + 97.5\ POLR + 101.5\ BUTR - (89 + (1 \times 4))\ SPRR + 4\ L \geq 0\ ,$$

	RLVR	LSRR	LCNR	HCNR	POLR	BUTR	SPRR	L_1	L_2	L_3		RHS
COST	3.5	4.3	3.5	4.0	5.0	2.0	-4.6	0.09	0.09	0.09		= MIN
OCTR. O	75.5	86.5	91.5	87	95.5	97.5	-89	4	2	1		≥ O
REGB	1	1	1	1	1	1	-1					= O
Lead B							-1	1				≤ O
							-1		1			≤ O
							-1			1		≤ O

Figure 20. Matrix formulation for three-segment TEL approximation

	RLVR	LSRR	LCNR	HCNR	POLR	BUTR	SPRR	L	
COST	3.5	4.3	3.5	4.0	5.0	2.0	-4.6	.09	=Min.
OCTR:1	79.5	90.5	95.5	93	97.5	101.5	-93	4	≥0
OCTR:3	82.5	93.5	98.5	97	99	104.5	-92	1	≥0
REGB	1	1	1	1	1	1	-1		=0
LEADB							-3	1	≤0

Figure 21. Matrix formulation for two-level TEL approximation

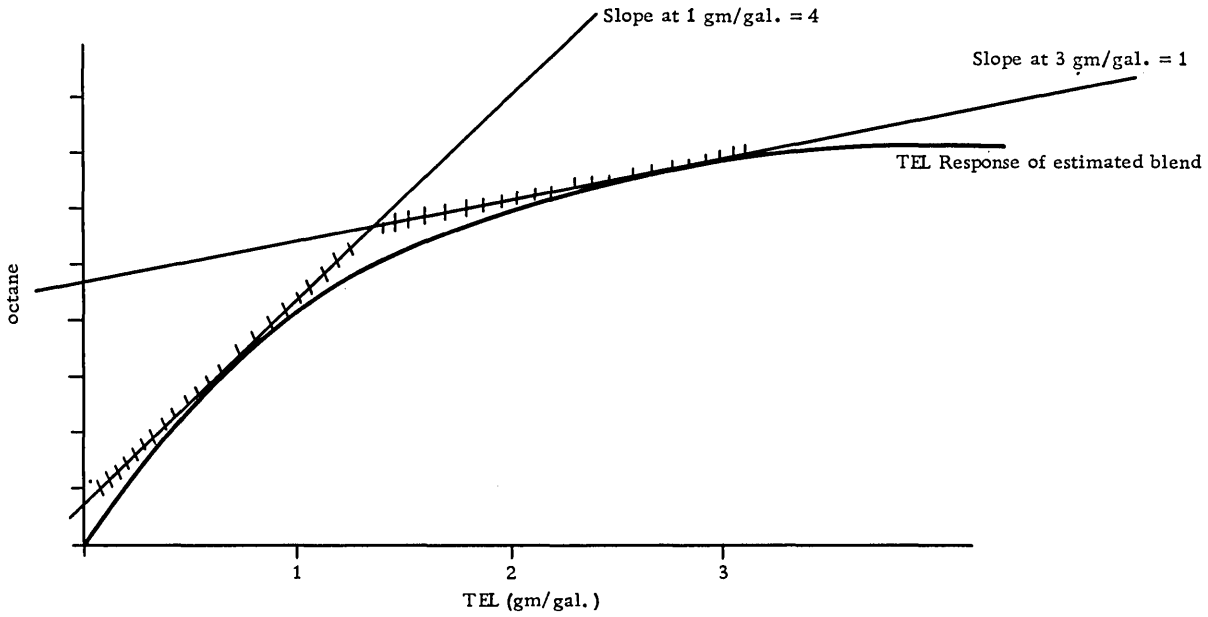

Figure 22. TEL response curve for two-level approximation

for octane at 1 gm/gal. where the slope (that is, rate of octane increase from additional TEL) is 4 (octane per gm/gal. TEL): and

$$82.5\ RLVR + 93.5\ LSRR + 98.5\ LCNR + 97\ HCNR + 99\ POLR + 104.5\ BUTR - (89 + (3 \times 1))\ SPRR + 1\ L \geq 0$$

for octane at 3 gm/gal. where the slope is 1.

The interesting parts of this formulation are the blend specification (SPRR) and lead (L) terms. In the first row, the tolerance-adjusted blend specification of 89 is increased by (1 gm/gal. TEL) times (rate of octane appreciation at 1 gm/gal. = 4). Since we have listed the components at 1 gm/gal. TEL levels, and we wish to determine how much TEL to add, we must indicate the impact of 1 gm/gal. TEL on the specification as well. The increase in the magnitude of the octane specification, in effect, cancels the increase in the magnitude of each component's octane number resulting from the addition of 1 gm/gal. TEL. Hence, the computation will indicate how much TEL is required to raise the octane of the components from zero TEL levels to the specified octane.

Similarly, the expression for octane at 3 gm/gal. TEL adds 3 times the rate of octane increase, or (3 gm/gal. TEL) times (octane appreciation at 3 gm/gal. = 1). The first expression assumes that octane increases by 4 for each gm/gal. TEL

added. The value of L (in gm TEL/gal./bbl.) will be determined on a maximum profit basis, again within the errors inherent in the approximations used.

A final formulation bounds the total TEL and ensures that no more than 3 gm/gal. TEL is used:

$$L/3 \leq SPRR$$

$$L - 3\ SPRR \leq 0$$

where L is TEL in gm/gal./bbl. and SPRR is total blend produced in bbls.

This two-level approximation method is more sensitive than the first method, because the latter depends on an unalterable estimate of the final blend TEL response curve, while this method computes two points actually on the blend TEL response curve. Thus, in the first method, both the TEL response curve and the slope of the chords joining the key points on that curve introduce error, while in the second method, since the points on the curve are computed, only the estimated slopes may introduce error.

A third method for formulating a linear approximation of TEL susceptibility (developed by Kawaratani, et al. — see reference 4) provides even more sensitivity to the lead response curve as the blend composition varies. In this method, we first plot the octane of each component at 0 and 3 gm/gal. TEL (Figure 23).

The envelope formed by joining the points defines the feasible area for the octane of any blend of these components. (Since LSRR and LCNR lie inside the feasible area, we need not employ them to form the feasible envelope.) Now, if we wish to meet a specific octane specification, say 90, we can overlay a chart of TEL-octane lines as in Figure 24. Each of the TEL-octane lines represents the quantity of TEL in gm/gal. required to raise the octane to 90. Thus, the illustration in Figure 24 indicates that material A (with 85 octane at 0 TEL and 90 octane at 3 gm/gal. TEL) requires 3.0 gm/gal. TEL to reach 90; material B (with 87 octane at 0 TEL and 93 octane at 3 gm/gal. TEL) requires 2.5 gm/gal.; material C (with 88 octane at 0 TEL and 96 octane at 3 gm/gal. TEL) requires 1.5 gm/gal.; and, of course, material D (with 90 octane at 0 TEL requires no additional TEL.

If we superimpose this 90-octane TEL chart over the envelope defined by the components of our blend, we obtain the configuration in Figure 25. Given this configuration, we can choose a number of points within the feasible envelope and also within the TEL-90-octane mesh which, in effect, defines a second envelope within which a feasible blend with a quality of 90 octane can be produced. We can, then, through a series of algebraic manipulations, make the new envelope (defined by the points x_1, x_2, x_3, x_4, x_5) a model of the original envelope and solve for a blend in the much smaller feasible area of the new model.

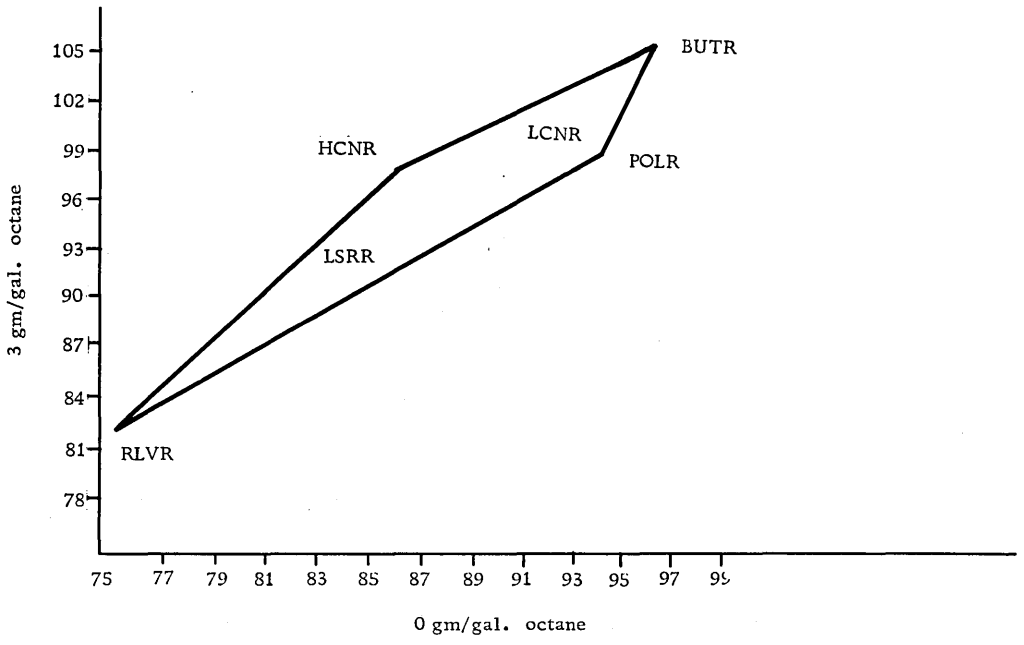

Figure 23. Envelope defining feasible octane area

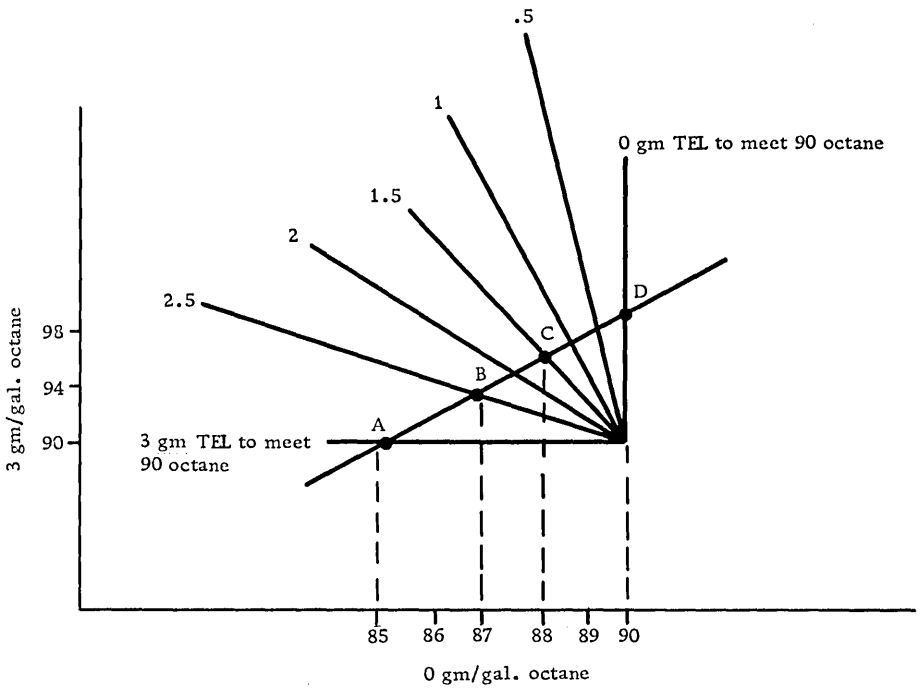

Figure 24. TEL "mesh" overlay chart for 90-octane specification

The advantage is that the mesh points within the superimposed TEL chart enable us to approximate TEL requirements quite accurately. The algebraic formulation consists of the following expressions:

$$75.5\ RLVR + 86.5\ LSRR + 91.5\ LCNR + 87\ HCNR + 95.5\ POLR + 97.5\ BUTR - 85.5\ X_1 - 86.5\ X_2 - 88.5\ X_3 - 87.5\ X_4 - 86.5\ X_5 = 0$$

at 0 TEL, and

$$82.5\ RLVR + 93.5\ LSRR + 98.5\ LCNR + 97\ HCNR + 99\ POLR + 104.5\ BUTR - 91\ X_1 - 92.8\ X_2 - 95.3\ X_3 - 92.8\ X_4 - 91\ X_5 = 0$$

at 3 gm/gal. TEL.

The following two material balance equations are also required, to equate the two envelopes.

$$RLVR + LSRR + LCNR + HCNR + POLR + BUTR - SPRR = 0$$

$$X_1 + X_2 + X_3 + X_4 + X_5 - SPRR = 0$$

We can now sum the actual quantities of lead which must be added to each of the model components in order to achieve a 90-octane blend:

$$2.6\ X_1 + 1.7\ X_2 + 0.4\ X_3 + 1.5\ X_4 + 2.5\ X_5 \leq -L\ \ 0.$$

The last formulation is an inequality because, conceivably, a maximum profit blend might give away one type of octane produced by TEL in order to meet some other octane specification. The matrix formulation for the mesh-point method is shown in Figure 26.

A number of implied restrictions are hidden in this structure. For instance, no final blend can have a 0-gm TEL octane number greater than 88.5 or less than 85.5 since these are the outer limits established by the model envelope. Similarly, and for the same reason, no final blend can have a 3-gm TEL octane number greater than 95.3 or less than 91. Further, nothing is gained if, in an attempt to increase accuracy, more than two mesh points are put on the same horizontal line, since only the end points can be used in the LP solution.

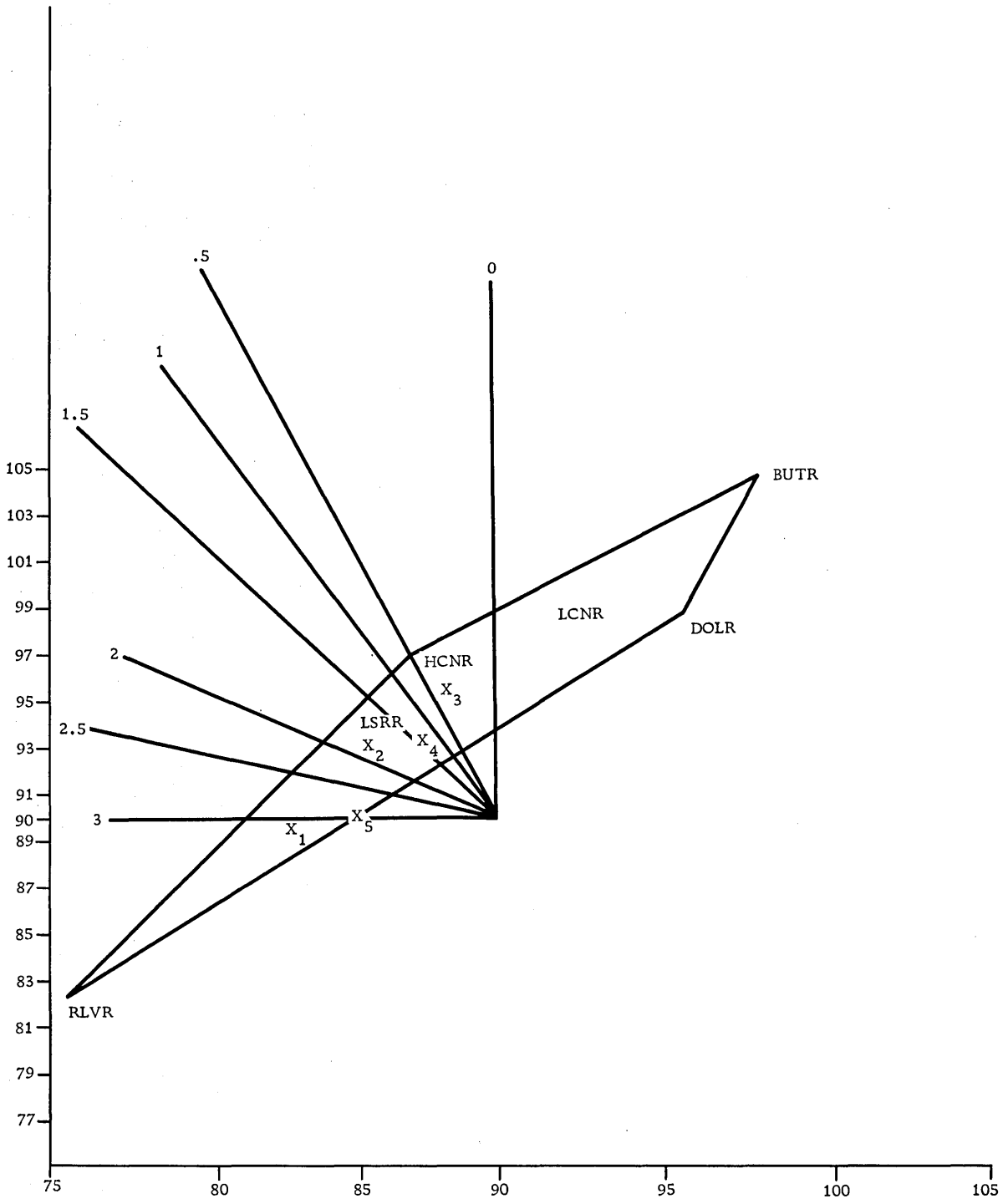

Figure 25. Application of overlay chart to envelope

	RLVR	LSRR	LCNR	HCNR	POLR	BUTR	SPRR	X_1	X_2	X_3	X_4	X_5	L	
COST	3.5	4.3	3.5	4.0	5.0	2.0	-4.6	0	0	0	0	0	.09	=MIN
OCTR:0	75.5	86.5	91.5	87	95.5	97.5		-85.5	-86.5	-88.5	-87.5	-86.5		=0
OCTR:3	82.5	93.5	98.5	97	99	104.5		-91	-92.8	-95.3	-92.8	-91		=0
REGB	1	1	1	1	1	1	-1							=0
MODB							-1	1	1	1	1	1		=0
LEADB								2.6	1.7	.4	1.5	2.5	-1	≤0

Figure 26. Matrix formulation for 90-octane mesh-point TEL approximation

In day-to-day gasoline blending models, when the lead response is fairly well known and constant, the first of the three methods just discussed is probably best. For the general case, the second method is preferred. The mesh-point method is recommended only if the blend composition is entirely unpredictable, and hence, the blend lead response curve is unavailable.

LINEAR FORMULATION OF COMPOSITE QUALITY SPECIFICATIONS

Often some quality specifications for gasoline blending can be made dependent on each other. For example, the table of Figure 27 relates maximum volatility at 158°F to Reid vapor pressure. This table is graphed as a step function in Figure 28. The cross-hatched line serves as an adequate linear approximation and is expressed by three linear inequalities which provide the limiting independent specifications for the percentage off at 158°F (denoted by R158X), the RVP, and the relationship between those two quality levels:

$$R158X \leq 33$$

$$RVP \leq 12$$

$$5 \times RVP + R158X \leq 86.5$$

RVP (tolerance 0.3)	Maximum % Off at 158° F (tolerance 3)
Below 10.8	33
10.8-11.0	32.5
11.0-11.2	31.5
11.2-11.4	30.5
11.4-11.6	29.5
11.6-11.8	28.5
11.8-12.0	27.5

Figure 27. Relation of maximum volatility at 158°F to Reid vapor pressure

In the model matrix, the usual adjustment for blending tolerances would be made in the individual expressions; for example, 158° ≤ 33 - 3 = 30, or RVP ≤ 12 - 0.3 = 11.7, but the adjustment for the composite specifications is less obvious. Frequently, one simply uses the larger tolerance involved. In this example, we would have 5 x RVP + R158X ≤ 86.5 - 3 = 83.5. (It is not appropriate to add tolerances since there is small probability of both qualities being off a large amount in the same direction at the same time.)

LINEAR FORMULATION OF PROCESS YIELD COMPONENTS

Particularly when considering the value and severity of reforming required to meet gasoline blending requirements at least cost, it is appropriate to incorporate the reformate yields in the gasoline blending model, rather than to run case studies with different availabilities. For example, if it takes 1.1 barrels of feed to produce a barrel of low severity reformate, and 1.2 barrels to produce a barrel of high severity reformate, the material balance for reformer feed (in a three-blend model) would be:

$$RFDR + RFDP + RFDS + 1.1\ LSRR + 1.1\ LSRP + 1.1\ LSRS + 1.2\ HSRR + 1.2\ HSRP + 1.2\ HSRS \leq RFDA$$

where:

RFDR = reformer feed blended to regular, MB/D

RFDP = reformer feed blended to premium, MB/D

RFDS = reformer feed blended to super, MB/D

LSRR = low severity reformate blended to regular, MB/D

LSRP = low severity reformate blended to premium, MB/D

LSRS = low severity reformate blended to super, MB/D

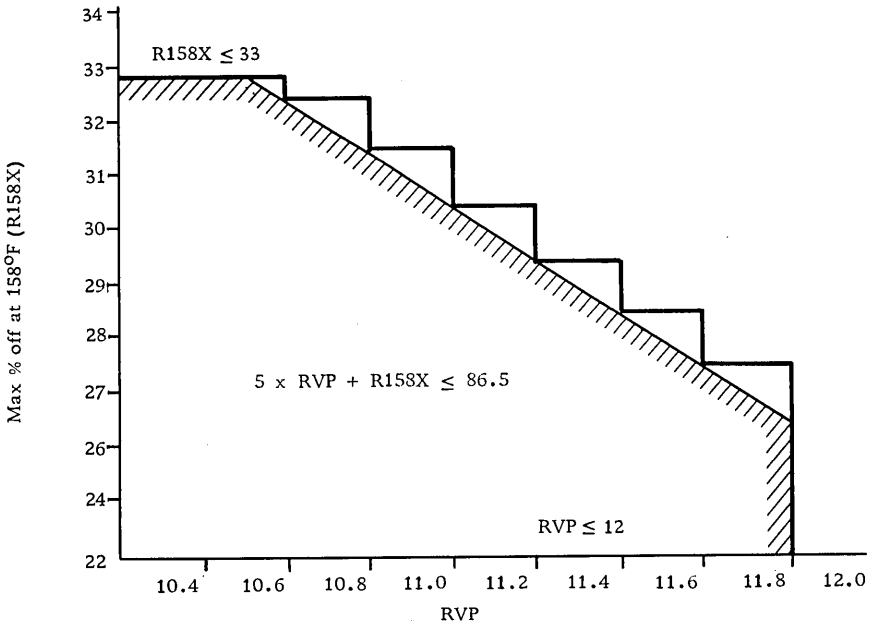

Figure 28. Linear approximation of the relation of volatility to Reid vapor pressure

HSRR = high severity reformate blended to regular, MB/D

HSRP = high severity reformate blended to premium, MB/D

HSRS = high severity reformate blended to super, MB/D

RFDA = reformer feed availability, MB/D

SAMPLE PROBLEM--THREE-BLEND MODEL

We shall now describe a three-blend LP model matrix which demonstrates the incorporation of refinements discussed in the preceding section--these refinements include linear approximations of TEL response (second method), composite quality specifications, and process yield components. In formulating the three-blend model, we use essentially the same components employed in the two-blend sample problem (Figure 6). Figure 29 is an engineering worksheet providing the essential input data. In addition to component characteristics and prices, the worksheet contains the following:

- Tetraethyl lead response for 0 and 3 gm/gal. in terms of both research octane and motor octane
- Reformer feed to low and high severity reformate relationship
- Specific minimum and/or maximum yield requirements for each blend
- Composite specification for maximum percentage off at 158°F and RVP
- Composite specification for minimum percentage off at 215°F and 240°F.

Figure 30 is the model matrix for this three-blend problem. It incorporates the two composite specifications, the tetraethyl lead formulation, and the process yield formulations. In the submatrix devoted to the regular blend, the second constraint row (RF51X) expresses the composite specification for RVP and maximum percentage off at 158°F listed in Figure 29. In the same submatrix, the constraint row designated RM23N provides the composite specification for minimum percentage off at 215°F and minimum percentage off at 240°F. The equivalent composite specifications for premium and super are incorporated into the appropriate submatrices as rows PF51X, PM23N, SF51X, and SM23N.

The tetraethyl lead formulation is contained in four rows within each submatrix. For the regular blend, RRONO RRON3 establish research octane at 0 and 3 gm/gal. TEL levels, while RMNO and RMON3 establish motor octane at 0 and 3 gm/gal. TEL levels. For each of these constraint rows, the LRR column indicates the rate of octane appreciation for regular. The last row in the regular blend submatrix (TELR) ensures that no more than 3 gm/gal. of TEL will be used in the blend. The same formulation is repeated for the premium and super blend submatrices.

Finally, the model matrix demands that no more than 40 MB/D of regular blend, 20 MB/D of premium, and 10 MB/D of super be produced. These limitations are incorporated as bounds on the column variables SPRR, SPPP, and SPSS, respectively. For convenience, the bounded variables are established as negative terms, and as a consequence, all

Components	Name	Max. Avail.	Cost	Max. RVP	Max. 158	Min. 215	Min. 240	Min. 300	Min. 350	Min. RO	Min. R3	Min. MO	Min. M3
Sour light virgin	RLV	3.8	3.5	6.2	7.5	38	77	95	100	78	81	71	74
Sweet light virgin	WLV	15.3	3.5	7.3	9.5	42	84	100	100	84	90	79	86
Special light virgin	SLV	10.4	3.5	12.1	20.0	54	88	100	100	76	90	72	89
Reformer feed	RFD	7.3	4.0	1.5	-7.0	12	36	60	95	60	76	68	77
Low severity reformate	LSR	1.1*	4.3	6.0	-7.0	19	35	56	89	86	95	78	87
High severity reformate	HSR	1.2*	4.55	6.2	-5.0	19	35	56	89	95	102	85	92
Pentane (cat)	PEN	3.54	2.5	16.0	83.0	118	110	100	100	96	100	83	87
Light cat naphtha	LCN	13.4	3.5	4.5	11.0	50	74	88	100	92	99	79	85
Heavy cat naphtha	HCN	7.6	4.0	0.0	-27.0	-20	-7	20	50	87	95	79	84
Alkylate	ALK	4.0	7.0	4.5	10.0	27	97	100	100	94	105	94	102
Polymer	POL	1.9	5.0	8.0	7.0	25	62	87	95	98	101	84	86
Butane	BUT	-	2.0	72.0	130.0	101	100	100	100	98	105	91	102
Tetraethyl lead to reg.	LRR	3gm/gal.	0.09							4.2	1.2	4.1	1.1
Tetraethyl lead to prem.	LPP	3gm/gal.	0.09							4.0	1.0	4.0	1.0
Tetraethyl lead to super	LSS	3gm/gal.	0.09							3.8	0.8	4.0	1.0
Requirements													
Regular blend	SPRR	40	4.50	11.7	30	40	60	-	91	89.0	92.6	82	85.3
Premium blend	SPPP	20	5.20	11.6	26	47	70	90	-	96	99.3	88.0	91.0
Super blend	SPSS	10	6.00	11.6	25	51	75	91	-	100.0	102.4	91	94.0

Composite and Other Specifications		Regular	Premium	Super
5 x RVP + Max. 158	≤	84.5	79	78
Max. 215	≤	49	54	-
2 x Min. 215 + 3 x Min. 240	≥	266	310	335

*These numbers are ratio factors rather than availabilities. They are explained in the section on linear formulation of process yield components.

Figure 29. Engineering worksheet for three-blend model

the coefficients in the three blend-produced columns, which in the small sample problem were negative, are positive in this formulation. They have, in effect, been multiplied by -1.

Figures 31 and 32 reproduce the basis variables and slacks reports, respectively, obtained from a solution of the three-blend model matrix (Figure 30). The solution indicates that maximum profit ($70, 300 per day) will be realized from a distribution of components resulting in 40 MB/D of regular blend, 15.165 MB/D of premium blend, and 10 MB/D of super blend. The information provided in the two reports can be further analyzed by the tabulation method illustrated in Figures 10 and 11.

CONCLUSION

This manual has been concerned with the basic gasoline blending process. A successful model of that process incorporates fundamental data -- the quality and availability of blend components, specifications, and costs and selling prices. Once constructed, the simple model may readily be expanded to make it more sensitive, and hence more accurate. The introduction of TEL formulations, composite specification constraints, and process yield considerations improves the model. As dynamic stock-processing relations are incorporated to replace static component availabilities, the model is made more realistic and versatile -- ultimately it may become a comprehensive refinery planning model with a detailed gasoline blending section.

Further, once constructed, the model can be used (with appropriate changes in specifications) to forecast seasonal inventory requirements, and hence serve to establish the optimum interseasonal storage levels of blending components.

A number of gasoline blenders have found that use of the basic blending matrix, without the process yield refinements, can result in profit increases of sufficient magnitude to justify the standard application of LP techniques. Blending matrices which are immediately applicable can later be expanded to reflect process variations as experience dictates.

	RLVR	WLVR	SLVR	RFDR	LSRR	HSRR	PENR	LCNR	HCNR	ALKR	POLR	BUTR	SPRR	LRR	RLVP	WLVP	SLVP	RFDP	LSRP	HSRP	PENP	LCNP	HCNP	ALKP	POLP	BUTP	SPPP	LPP	RLVS	WLVS	SLVS	RFDS	LSRS	HSRS	PENS	LCNS	HCNS	ALKS	POLS	BUTS	SPSS	LSS	
COST	3.50	3.50	3.50	4.00	4.50	4.95	2.00	3.50	4.00	7.00	5.00	2.00	4.50	.09	3.50	3.50	3.50	4.00	4.50	4.95	2.00	3.50	4.00	7.00	5.00	2.00	5.20	.09	3.50	3.50	3.50	4.00	4.50	4.95	2.00	3.50	4.00	7.00	5.00	2.00	6.00	.09	= MIN.
RLVA	1														1														1															≤ 3.8
WLVA		1														1														1														≤15.3
SLVA			1														1														1												≤10.4
RFDA				1	1.1	1.2												1	1.1	1.2											1	1.1	1.2								≤7.3
PENA							1														1														1							≤3.54
LCNA								1														1													1							≤13.4
HCNA									1														1													1						≤7.6
ALKA										1														1													1				≤4.
POLA											1														1													1			≤1.9
RRVPX	6.2	7.3	12.1	1.5	6	6.2	16	4.5	0	4.5	8	72	11.7																													≤0
RF51X	38.5	46	80.5	0.5	25	26	163	33.1	27	32.5	47	490	84.5																													≤0
R158X	7.5	9.5	20	-7	-7	-5	83	11	-27	10	7	130	30																													≤0
R215N	38	42	54	12	19	19	118	50	-20	27	25	101	40																													≥0
R215X	38	42	54	12	19	19	118	50	-20	27	25	101	49																													≤0
R240N	77	84	88	36	35	35	110	74	-7	97	62	100	60																													≥0
RM23N	307	336	372	132	143	143	566	322	-61	345	236	502	266																													≥0
R350N	100	100	100	95	89	89	100	100	50	100	95	100	91																													≥0
RR0N0	78	79	76	60	86	95	100	92	87	94	98	98	89	4.2																												≤0
RR0N3	81	84	90	76	95	102	100	99	95	105	101	105	92.6	1.2																												≤0
RM0N0	71	90	72	68	78	85	83	79	79	94	84	91	82	4.1																												≤0
RM0N3	74	86	89	77	87	92	87	85	87	102	86	102	85.3	1.1																												≤0
RIBAL	1	1	1	1	1	1	1	1	1	1	1	1	1																													=0
TELR													3	1																												≤0
PRVPX															6.2	7.3	12.1	1.5	6	6.2	16	4.5	0	4.5	8	72	11.6															≤0
PF51X															38.5	46	80.5	0.5	25	26	163	33.1	27	32.5	47	490	79															≤0
P158X															7.5	9.5	20	-7	-7	-5	83	11	-27	10	7	130	26															≤0
P215N															38	42	54	12	19	19	118	50	-20	27	25	101	47															≥0
P215X															38	42	54	12	19	19	118	50	-20	27	25	101	54															≤0
P240N															77	84	88	36	35	35	110	74	-7	97	62	100	70															≥0
PM23N															307	336	372	132	143	143	566	322	-61	345	236	502	310															≥0
P300N															95	100	100	60	56	56	100	88	20	100	87	100	90															≥0
PR0N0															78	79	76	60	86	95	96	92	87	94	98	98	98	4.0															≤0
PR0N3															81	84	90	76	95	102	100	99	95	105	101	105	102.8	1.0															≤0
PM0N0															71	90	72	68	78	85	83	79	79	94	84	91	88	4.0															≤0
PM0N3															74	86	89	77	87	92	87	85	87	102	86	102	91	1.0															≤0
PIBAL															1	1	1	1	1	1	1	1	1	1	1	1	1																=0
TELP																											3	1															≤0
SRVPX																													6.2	7.3	12.1	1.5	6	6.2	16	4.5	0	4.5	8	72	11.6		≤0
SF51X																													38.5	46	80.5	0.5	25	26	163	33.1	27	32.5	47	490	78		≤0
S158X																													7.5	9.5	20	-7	-7	-5	83	11	-27	10	7	130	25		≤0
S215N																													38	42	54	12	19	19	118	50	-20	27	25	101	51		≥0
S240N																													77	84	88	36	35	35	110	74	-7	97	62	100	75		≥0
SM23N																													307	336	372	132	143	143	566	322	-61	345	236	502	335		≥0
S300N																													95	100	100	60	56	56	100	88	20	100	87	100	91		≥0
SR0N0																													78	79	76	60	86	95	96	92	87	94	98	98	100	3.8	≤0
SR0N3																													81	84	90	76	95	102	100	99	95	105	101	105	102.4	,8	≤0
SM0N0																													71	90	72	68	78	85	83	79	79	94	84	91	91	4.0	≤0
SM0N3																													74	86	89	77	87	92	87	85	87	102	86	102	94	1.0	≤0
SIBAL																													1	1	1	1	1	1	1	1	1	1	1	1	1		=0
TELS																																									3	1	≤0
													-40≤SPRR≤0														-20≤SPPP≤0													-10≤SPSS≤0		

Figure 30. LP model matrix for three-blend problem

OUTPUT
BASIS. MANTISSA 10 TOLERANCES 08 05 03 03 02

VARBLS	NAME	ACTIVITY LEVEL
	RLVR.	3.800
	WLVR	15.300
	SLVR	8.533
	RFDR	.717
	PENR	1.735
	HCNR	7.128
	BUTR	2.787
	SPRR	40.000-
	LRR	54.060
	SLVP	.500
	HSRP	.259
	PENP	.403
	LCNP	11.012
	ALKP	1.034
	POLP	.623
	BUTP	1.334
	SPPP	15.165-
	LPP	45.496
	SLVS	1.367
	HSRS	1.017
	PENS	1.402
	LCNS	2.388
	ALKS	2.127
	POLS	1.277
	BUTS	.422
	SPSS	10.000-
	LSS	30.000

Figure 31. Basis variables report -- three-blend problem

SLACKS	TYPE	NAME	ACTIVITY LEVEL	SIMPLEX MULT.
	F	COST	70.300-	
	+W	RLVA		.797
	+W	WLVA		.886
	+W	SLVA		.530
	+F	RFDA	5.052	
	+W	PENA		1.914
	+W	LCNA		1.641
	+F	HCNA	.472	
	+F	ALKA	.839	
	+W	POLA		.255
	+W	RRVPX		.043
	+F	RF51X	1.751	
	+F	R158X	546.679	
	-W	R215N		.002-
	+F	R215X	360.000	
	-F	R240N	374.136	
	-F	RM23N	882.409	
	-W	R350N		.013-
	-W	RRONO		.021-
	-F	RRON3	11.429	
	-F	RMONO	44.011	
	-F	RMON3	93.085	
	G	RBAL	.000	1.529-
	+F	TELR	65.940	
	+F	PRVPX	6.583	
	+W	PF51X		.011
	+F	P158X	42.912	
	-F	P215N	95.524	
	+F	P215X	10.633	
	-F	P240N	123.038	
	-F	PM23N	469.169	
	-W	P300N		.041-
	-F	PRONO	99.190	
	-W	PRON3		.135-
	-F	PMONO	79.793	
	-W	PMON3		.032-
	W	PBAL		14.192
	+W	TELP		.077
	+F	SRVPX	9.797	
	+W	SF51X		.011
	+W	S158X		.000
	-W	S215N		.000
	-F	S240N	64.536	
	-F	SM23N	113.608	
	-W	S300N		.041-
	-F	SRONO	35.239	
	-W	SRON3		.135-
	-F	SMONO	45.496	
	-W	SMON3		.032-
	W	SBAL		14.231
	+W	TELS		.050

Figure 32. Slacks report -- three-blend problem

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