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MANIPULATOR DESIGN VIGNETTES

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This memo is about mechanical arms. The literature on robotics seems to be deficient in such discussions, perhaps because not enough sharp theoretical problems have been formulated to attract interest. I'm sure many of these matters have been discussed in other literatures -- prosthetics, orthopedics, mechanical engineering, etc., and references to such discussions would be welcome. We raise these issues in the context of designing the "mini-robot" system in the A. I. Laboratory in 1972-1973. But we would like to attract the interest of the general heuristic programming community to such questions.

1. Degrees of freedom: serial vs. parallel?

A manipulator needs many degrees of freedom. To understand the resulting complications, we need some sort of general framework for analysis, but we don't even have a useful taxonomy for classifying the problems. Here we will contrast two extreme kinds of anatomical approaches; quite a few issues emerge clearly even from this primitive dichotomy. The two extremes are "serial" and "parallel".

In a SERIAL arm, we have a sequence of bodies, usually rigid rods, whose positions are determined by a series of constraints -- joints -- in which each constraint fixes the position of a body that serves as the support for all its successors. Thus the degrees of freedom have a distinct order.

All the industrial manipulators are of this character. The human arm is representative of the serial variety, in general plan, although it is really an intermediate case because each joint may combine two or even three degrees of freedom. The human wrist, from the outside, appears to have three degrees, but the rotations really associated with the radius-ulnar relation which should be considered a separate joint.

The PARALLEL concept is perhaps best illustrated by the way an animal's body is supported by its legs. Here several constraints simultaneously determine the relation of one body to another (the ground, in this case). A clearer example is that of a crane or antenna mast supported by guy wires. Because six constraints are needed to fix an object in three dimensions, that bounds the amount of parallelism possible at each "joint".

The sequential succession of constraints in the serial arm causes many problems that seem to grow in a multiplicative way:

Errors and uncertainties are cascaded and cumulative.

Rigidity is relatively low because of long moment arms.

Inertial and gravity effects are large, especially if the proximal joints have to support the mass of the distal motors. This and the low rigidity combine to produce large, annoying, low frequency vibrations. These make precise positioning slow (to avoid dangerous overshoots) and make delicate force-feedback measurements impractical.

Power supply and "innervation" require complicated "threading" through or around joints, especially if the motors are not built in, but work through tendons.

Some of these problems can be reduced by making the proximal joints much larger than the distal ones, but this yields a massive and clumsy system, viz., a milling machine.

All these problems can be controlled to various degrees, given enough attention to weight and dynamics, as in the vertebrate and insect arms. More dramatic illustrations of serial cascading of joints are seen in vertebral columns, especially of the serpent. Strictly speaking, none of these are purely serial, each having two or more actuators for each joint. This is one way to reduce the amount of serialization.

Another way to reduce the amount of cascading is to divide the mobility of the whole system into two parts by moving the work as well as the manipulator. Example: In some milling-machines that need three degrees of freedom, the bed on which the work is mounted has two horizontal axes; the third axis moves only the cutting-head. Another "solution" is the moving-robot approach: by walking or driving about the floor, one gets two degrees of freedom without the cost of transmitting power and information through two joints. The machine has to be attached to the floor anyway, one can argue, so why not exploit this? One can regard the floor as a ball-joint with zero curvature!

In any case, the parallel approach promises to reduce complication. Ball joints, with three degrees of freedom, have no more parts than single hinge joints. What really is difficult is to make a two-degree joint, with freedom to bend but stiff axially; this is the "universal joint" problem.

1.1 Parallel Manipulators

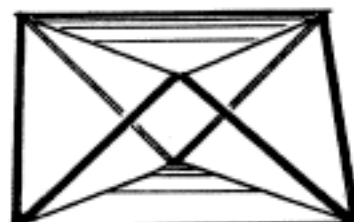
The parallel approach is complementary, with quite different problems and features. To introduce it, begin by considering the general problem of positioning a rigid body in space relative to a frame of reference.

Suppose given three fixed reference points in space and three points on the rigid body. Then there are nine distance-pairs between these points, but specifying almost any six of these causes the other three to be determined. This observation suggests a very simple and direct approach to design. Choose six of these distances. Then construct a manipulator by realizing physically each length constraint as some mechanical length-determining device, such as a hydraulic cylinder or other "linear actuator".

Each actuator must be terminated by a spherical joint or elastic coupling. Then we achieve complete mechanical control of the rigid object simply by instructing separately the motors for all degrees of freedom, without cascading their effects.

A particularly symmetrical example of such a design is shown below: we choose six edges of a cube to position one equilateral triangle with respect to another. The resulting structure outlines an octahedron:

The six edges are the ones that form the hexagonal shadow when a cube is balanced on one of its vertices!



In principle, it follows from the basic constraint-determination idea, that one can position the body in any location and orientation in three-dimensional space. However, there are practical problems:

In certain positions the constraints degenerate (e.g., two rods coincide) making the position indeterminate. The structure will flop.

As we approach a degenerate situation, the mechanical advantages become too small, great forces are needed, and the structure will buckle.

In practical implementations, it is hard to find large ranges in which the supports do not interfere with one another by colliding. This is particularly severe in regard to axial rotations.

The supports must subtend a substantial space angle to get much stiffness. On the other hand, an arm should subtend a small space angle (looking from the hand) to achieve dexterity. In optical jargon, one wants a hand to have a large "f-number"!

The ability to get around obstacles is limited. With a serial arm one can use redundant degrees of freedom to get a tentacular effect. It is an illusion, however, that serial arms inherently provide dextrous access, since unless the elbows are redundant, the multi-jointed arm doesn't allow for alternative paths. Of course, with six degrees of freedom one has a three-dimensional selection of ways to reach a point in the workspace, and one can trade between global arm-path configuration and the precise direction of arrival at the work.

The actuators must be length-changers, in contrast to those in the serial arm, which can be linear or angular. This is usually a large advantage, because in the parallel system, the natural way to effect constraints is by pulling tendons. Cable-pulling mechanisms are easy to design and can be very compact.

Although parallel systems have disadvantages, these seem quite different from those of the serial system. There are important advantages, as well.

Very great simplicity, because of the non-interactions.

The instrumentation of force-sensing is particularly convenient; the longitudinal stresses in the supports determine completely the forces operating on the mobile plate (unfortunately, including gravity). As shown in 1.3 below, one can exploit this to get very elegant and useful kinds of force- and tactile- feedback.

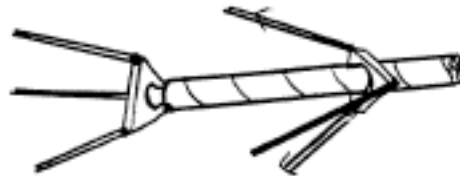
The forces are entirely axial, so that light, thin tubes can yield great strength. Serial arms in general, and angular actuators in particular, have bad weight-strength characteristics.

1.2 Problems:

Design a geometry that gives better rotational mobility.

Can one separate the degrees into two sections, which engage a ball-screw and nut pair, to get extensive rotation? Is there a simple attachment linkage that gets such an effect?

Each plate can be located anywhere in space. The two locations determine the screw location, and their distance apart determines the screw rotation.

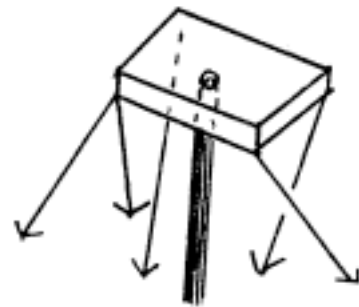


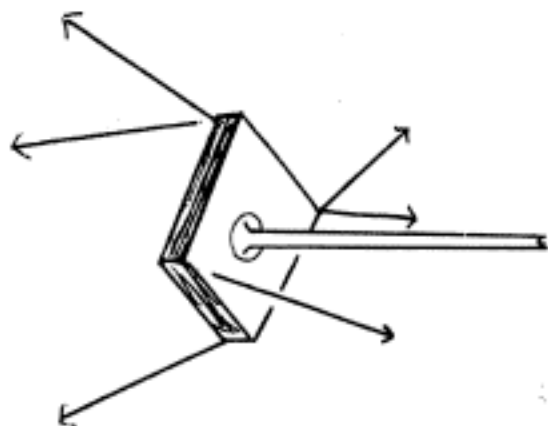
There are less symmetrical parallel configurations that are easier to understand. One can separate out three axes for location, two for orientation, and the sixth for axial rotation. Is this a good idea? It might make heuristic planning easier in some situations, but that would not justify realizing that decomposition in the hardware.

Idea: stack up two or even three octahedral sections. If one could control all the resulting motions, one would have a highly dextrous "orm"- or snake- like system, but let us agree not to consider such complexities. However, suppose one could invent a linkage or related arrangement that duplicates the constraints in each section. This would double or triple the rotational mobility. It would also impart a helical character to the motion, which would probably be more a nuisance than a virtue. Such an invention, if not too flimsy, would eliminate many objections to the simple octahedral system. Perhaps control of the more complex, excess-degree-of-freedom version could be done by an application of the "fourier-analysis" concept discussed below.

How can one design such a system with countersprings or counterweights to cancel gravity, to permit more sensitive force sensing?

Consider tension-extension trades. One system would have a single incompressible rod (with extension motor) with the other five degrees of freedom constrained and actuated by cables, as in an antenna guy-wire arrangement, or as in a building-crane. Cable pullers are much simpler and lighter than linear compression actuators; all one needs is a motor and a spool.

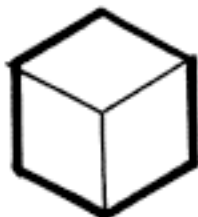




Finally, one might consider geometries in which ALL constraints are tension. The simplest version of this makes anchor points span more than a half-plane about the workspace, which is a nuisance. But the hand can easily be cantilevered to be outside the convex hull of the support points, as shown here. Although we discussed tripod-based geometries above, there is no reason the six constraints should not be anchored from four, five, or six space points. One needs at least four for the all-tension model. With redundant cables, one could overcome some of the collision problems, at some cost in complication.

1.3 Can You Visualize This?

In the octahedral geometry proposed above, the substructure of six rods, each meeting two others at right angles, is mechanically rigid, even though the rods are permitted axial rotational freedom at any point (but constrained to meet one another at right angles). However, there is another such "circuit" of six rods:



RIGID



NOT RIGID

This "linkage" is not rigid, but has one degree of freedom and can be contorted in a remarkably tortuous cycle in which each rod occupies every position in the structure. Thus, the rigidity of the "octahedral" form is a non-local feature of the structure. Can anyone give a simple explanation of this degeneracy? To experiment, you can make the structures out of six right-angle "BNC" type radar connectors or equivalent.

2. FORCE SENSING

I wish I understood better the issues concerning kinesthetic feedback. The advantages of "bilateral" servomotors over "open-loop" control are easily demonstrated, but my own experience is that the quality of the "feeling" one gets in operating a bilateral master-slave manipulator is very low. If the gain is high, the system is unstable, while if the gain is modest it feels mushy. Presumably the inertial forces, when reflected back to the human user, cannot be opposed quickly enough because of our 200 millisecond reaction time problems.

How does the human arm and control system manage so well? We have at least three levels of not-too-tightly coupled systems with sense and motor devices of different characters: motor-force (as in the bilateral servo manipulators), local strain sensing at the joint areas, and tactile sensing at the exterior surfaces. This information is used, then, in a heterarchical control structure.

How does this control structure work? I do not know the literature very well. I suspect that it is not well understood, and the control-theory analyses I have seen (but not studied) do not seem to get to the point. Presumably, the heterarchical system uses several different kinds of information, about position, pressure, speed, etc., in local systems each with its own loops and parameters. The control structure couples them by sending around signals that adjust gains, time-constants, etc. The higher levels computer these by combining state information with goal information.

What's wrong with the simple "bilateral servo" system? (This is the system that measures the forces reflected through the motors that actuate the joints.) One trouble is that the force on an exterior object is the SUM of the driving force and the inertial force of the moving system, as it is decelerated by pushing on the obstacle. The "force-reflecting" servo can never see the latter force. Forces at the hand depend on momentum terms from all the way up the arm. "Theoretically", the control system can contain a dynamic model of the arm, compute the inertial forces, and subtract them from the actual measurements, but this means measuring small differences between large numbers, and there is a lot of noise from friction, dynamical vibrations, and so on. The gravity forces have to be subtracted, too. Notice that in the serial arm this is not an all-or-none argument. The signal-noise ratio just mentioned is poor for the "upper" or "proximal" (near the arm origin) joints and good for the "distal" (near the hand) joints. For a non-redundant mechanism there is no sure way to guarantee that the sensitivity required for a delicate manipulation will be reflected to a suitably distal joint!

This argument shows a fundamental advantage, I think, for having a complete set of force-sensors in the wrist or very close to the workpoint. That is, for a system like that described in 2.1 below for measuring forces near the workpoint without the noise of large inertial components. With a wrist force-sensor, one knows instantly when the hand has made contact, and how hard. If the arm is moving rapidly, one cannot stop it instantly, but if the wrist can be relaxed while the upper arm decelerates, the contact can still be gentle.

What, by the way, is a relaxed wrist? In a non-redundant configuration one can have high compliance -- low-inertial gentle contact -- only along those dimensions whose actuators are close -- in the serial sequence -- to the hand. This is why one should give a high design priority to getting at least three degrees to combine in a recognizable wrist-like structure. Anthropomorphism has a lot going for it. But it is hard to put all mobility at the wrist, and even the human hand is vulnerable to thrust through the radial axis: as in catching a ball on the end of one's finger.

In any case, if a redundant wrist is relaxed, and the force sensors signal a collision, there may still be time to react before the pressure gets large; with a non-redundant bilateral servo, the poor signal-noise ratio requires us to wait until the force is larger.

If we cannot afford full motorization of the wrist, it would still be valuable to have a spring-loaded, lockable, compliant joint there.

The same arguments, in micro-form, apply to fingers as well, and dictate a third stage of sensing. This again might be a combination of joint-force elements and, of course, the ultimate dispersion of nerve-endings on the tactile surfaces.

2.1 Force feedback for position-sensing.

Speaking of tactile sensors, it is an amazing mathematical fact that, under certain conditions, one can use joint force-sensing (or even bilateral servos) to get the effect of tactile sensors over a surface. That is, the measurements can tell where a surface has been touched! The "certain conditions" are simple and somewhat restrictive; the surface must be touched at just one point. This is not so bad, because this usually happens at a "first contact". If the condition is not met, one still gets important information that can be useful when combined heterarchically with other knowledge. Let's first see how force-sensors can yield position information.

Consider a rigid bar supported by two force sensors A and B:



If we press at point "A" with unit force we will read

$$A = 1 \quad B = 0$$

If we press with unit force at "B" we will read

$$A = 0 \quad B = 1$$

If we press at "C" we will read (assuming A and B are one unit apart)

$$\begin{aligned} A + B &= 1 \quad (\text{total downward force}) && \text{AND} \\ 2A + B &= 0 \quad (\text{zero net torque}), && \text{hence,} \\ &A = -1 \quad B = 2. \end{aligned}$$

More generally, if "x" is the distance from A to C, and "F" is the force, we will have:

$$\begin{aligned} A + B &= F \\ AX + (B-1)x &= 0 \quad \text{or} \quad Fx - B = 0 \quad (\text{torque cancellation}) \end{aligned}$$

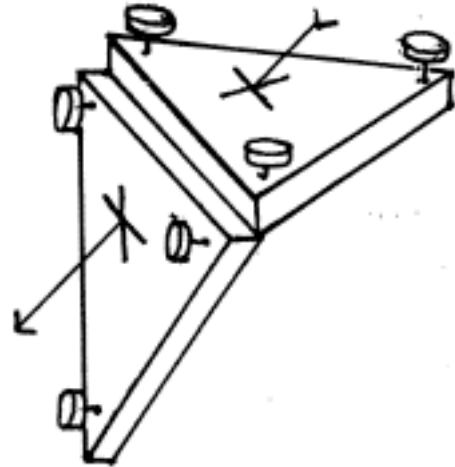
so that we get

$$X = B/(A+B)$$

showing that we can read from the two dials, almost directly, both the location of the force and its magnitude! By a similar argument (in 2.2 below) we will see that if a triangular plate is supported by three scales, we can again locate the point of application of an arbitrary force and its magnitude, even though it can now be anywhere in a plane.

Can we do better? A fixed body is subject to six constraints, so we can make three more measurements. We do this in 2.2 and show that with six such readings, one can determine not only the place and magnitude of the applied force but also its orientation and its axial torque. More precisely, we can determine the force vector up to lying on a definite line in space.

Here is a false argument to make this result seem plausible. Imagine joining two triangular plates, as above, at right angles. When we apply a force to this object, the readings on the six scales would determine two points, one in each triangle. One might be tempted to conclude that that the line of force is exactly the line determined by those two points.



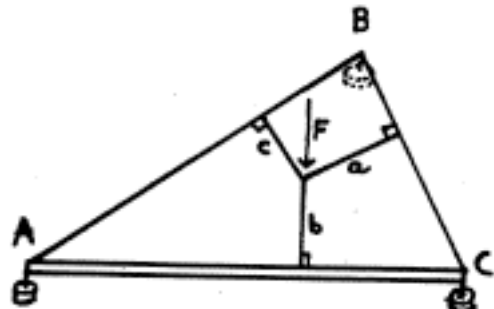
That argument is false because the scales' configuration doesn't constrain the body along the line of intersection, but it suggests why such a measurement might be possible. In 2.2 we carry out the analysis completely for a fully constrained configuration.

Before doing the calculation, we point out an important consequence. If we can determine the line of application of a force then we can determine the point of application of the force -- provided only that we know the shape of the surface! There is no reason to restrict our surface to be a particular plane; given any convex (and most any non-convex) shape, a space line enters the surface at a unique point. So the six-axis force measurement can serve as a tactile-position sensor over the entire surface!

2.2 Exact calculation of the Octahedral Strain-Gauge

We now show how to resolve the force on the "octahedral" arm by a sequence of easy superposition steps. This derivation illustrates the immense power of knowing a system is linear: it is just a sequence of superposition arguments.

First consider an equilateral triangular plate supported by three scales at A, B, and C. Let "a", "b", and "c" be the distances from an arbitrary force point to the corresponding opposite sides -- these are the so-called Barycentric Coordinates.

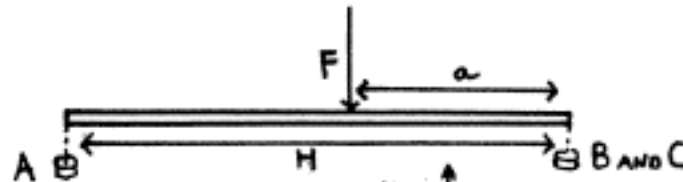


If F is the magnitude of the force, then $F = A + B + C$ because the system is in vertical equilibrium. And the three weights are in simple proportion to the three distances a , b and c . One can see this, for example, by viewing the scene in a plane orthogonal to the line BC :

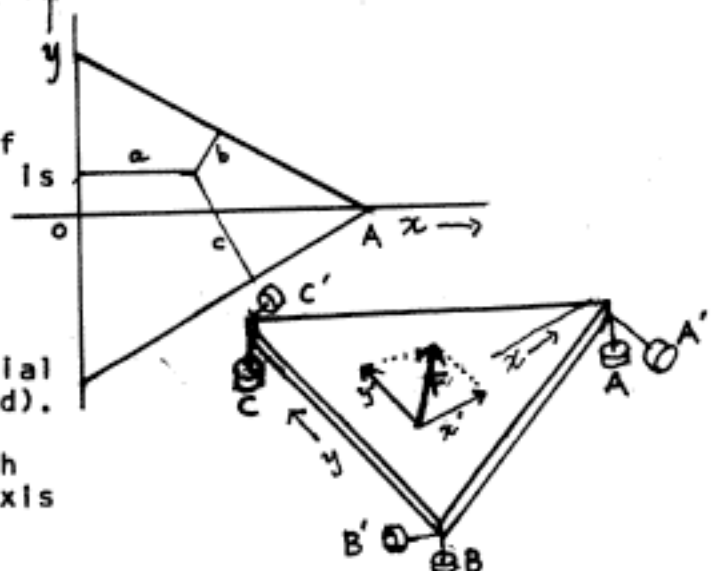
$$AH = Fa$$

$$BH = Fb$$

$$CH = Fc$$



where $aF = AH$, for example, simply because of cancellation of the two moments aF and AH . So it is easy to find the exact point of application of F , given these three measurements. It is convenient now to introduce some Cartesian Coordinates, and one can see by parallelograms of forces that $x = a$ and $y = \frac{\sqrt{3}}{4}(c - b)$



A force on the plate may have tangential components as well as normal (downward). We will detect these by adding three more strain gauges as shown here. Each senses differential motion along an axis parallel to a side of the triangle.

We have shown a tangential force F' lying IN THE PLANE, with components x' and y' . No matter where such a force is applied, equilibrium dictates that

$$x' = (\sqrt{3}/2)(C' - B')$$

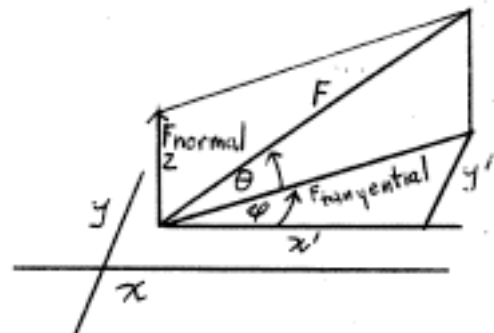
$$\text{and } y' = C'/2 + B'/2 - A'$$

So these three measurements give us the direction $\tan^{-1}(y'/x')$ and magnitude $A' + B' + C' - A'B' - A'C' - B'C'$ of the tangential force component. If the actual force has both normal and tangential components we note that $[A, B, C]$ still determines the point of application, since the three new gauges do not affect the normal measurements.

Thus from our knowledge of the tangential F' and the normal F , we get a complete three dimensional description of the total force F^* in azimuth-elevation coordinates:

$$\text{elevation } \theta = \tan^{-1}(A+B+C)/|F'|$$

$$\text{azimuth } \varphi = \tan^{-1}(C'+B'+2A')/x'$$



showing that the actual computation is not particularly complicated.

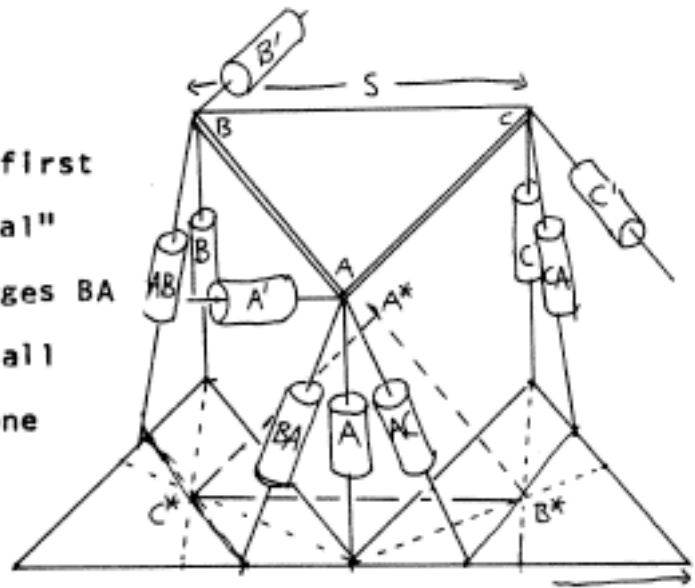
We derived only two quantities, φ and θ from the three measurements $|A', B', C'|$. What is the third dimension? We claim it is the tangential torque on the plate. To see this, assume that the strain-gauges' deflections are approximately linear in force, and consider the energy of distortion of the system regarded as supported by springs. Given a certain force, the energy in the tangential springs will be minimal when there is no torque, simply because an applied torque will always be resisted, hence will increase the system's energy, which is proportional to $A'^2 + B'^2 + C'^2$. For a pure y-force we have $B' = C'$ and $F' = C' - A'$. In this case $E = A'^2 + 2(A' + F')^2$ which is a minimum when $F' = -3A'/2$.

It follows that $0 = A' + B' + C'$.

A similar argument shows that a pure X-force also yields $A' + B' + C' = 0$. Hence, by superposition, since each term is a linear function of all force components, this sum is zero for any torque-free force. Furthermore, $A' + B' + C'$ is proportional to torque applied at the center, is invariant of all translation forces, and is the only remaining linear function that is so invariant, so it must be the torque -- assuming correctly that the torque is a linear function of the measurements.

In practical application of this calculation, one might assume that no tangential component of torque actually can be applied to the surface in ordinary manipulation. That would require a push-pull couple, and pulling requires glue. Normal components, however occur naturally at ordinary frictional surface contacts.

Now, finally, we return to the octahedral strain gauge. We will first replace our "normal" and "tangential" gauges A' and A by the oblique gauges BA and AC shown here. Note that now all four of A , A' , BA , and AC lie in one plane! Therefore:



$$\begin{aligned} A &= pBA + qAC \\ A' &= rBA + sAC \end{aligned}$$

by a simple planar change of coordinates. Now in our diagram, C^* is the base point for attaching two gauges in our initial octahedral configuration. Again we can use a planar configuration, for by choosing the indicated dimensions for d and $d/3$ we have arranged that all of A , B , C^* , AB , and BA lie in one plane. Hence, we can replace AB and BA by the correct pair BC^* and C^*A -- not shown in the diagram to avoid clutter -- by another planar transform:

$$\begin{aligned} BA &= p'BC^* + q'C^*A \\ AB &= r'BC^* + s'C^*A \end{aligned}$$

which completes the entire analysis. The entire transformation can be condensed into a 6x6 matrix:

$$A = p''BC^* + q''C^*A + r''AB^* + s''B^*C$$

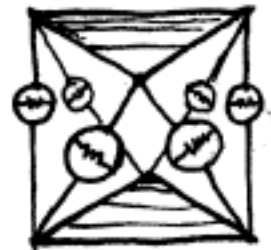
All this applies only to small distortions in the structure, as one would indeed have in a quasi-isometric strain-gauge. If one wants substantial motion, as one would in a bilateral force-servo in which the beams of the octahedral supports are also the manipulator's degrees of freedom, the coefficients would not be constant, and one would need trigonometrical approximations.

When I first considered the octahedral configuration, I asked Jayant Shah to analyse the forces, and he observed the theorem that one could resolve the force along a unique space-line. I had assumed that there was a lot of good information, but had not realized exactly how much. Shah also wrote down a matrix analysis (not using all these symmetry tricks) for resolving the forces. We cannot find any copy of Shah's brief report. Does anyone have one?

We decided to build a 6-axis wrist. Shah concluded that the symmetrical configuration might require excessively large dynamic range to represent forces and torques both adequately, and designed an unsymmetrical version that was actually built, with the help of Tom Callahan and John Roe. It is currently working, and (November 1972) has been programmed by David Silver to turn nuts on bolts in a strikingly natural and smooth procedure.

On completing the present analysis, I think it might be better to stick with the octahedral configuration. The ratio of height to depth (length to diameter) should allow selection of force-torque sensitivity ratio. The symmetric design is much easier to fabricate, and has excellent weight-strength properties because all the forces are along axial rods.

Here are two configurations that might be reasonable -- light and easy to make. The first is simply a perforated cylinder, with strain gauges along the remaining parts. The anchor-ring form can be as sensitive as one wants, and the strain gauge mountings do not interfere with the insertion of the struts: it might be very good for use with LVDT displacement sensors.



It is tempting to consider combining the force sensor and a position servo in each of the six beams, to make a mobile force sensing wrist. Of course, once the thing moves appreciably, the resolution of forces will require the more general calculation, with trigonometric coefficients. But this should not be a serious problem because I am sure that there are powerful computational short cuts for such configurations. See section 3.

I'm sure there are competitive designs. Six-axis force sensors are common in instrumentation. This one has a particularly nice clear central channel. A wrist really doesn't need great rigidity; later we make a case for the contrary. So the dominating design criteria might well be centered on making the sensors rugged and with great dynamic range.

3. Approximation and Interpolation

It is quite easy to program a numerically controlled milling machine to move to a specified space-position. The motion axes of such a machine are orthogonal, and so one has only to move each axis the required distance in Cartesian coordinates from where it is to where it should be.

But there has grown up a myth that, if an arm has a less regular geometry, things are so much more complicated as to pose critical difficulties. It is thus often suggested that there are serious problems in programming an arm with several rotary joints, or one with more degrees of freedom than are absolutely required. So prevalent is this belief that (we suspect) the mechanical design of advanced manipulators has been constrained accordingly.

What are the problems? There are different ways that a computation can be complicated. The mathematical solution might be opaque. Indeed, with a complex arm, the hand-position expressed as a function of the joint parameters is messy, involving a cascade of trigonometric matrices. Accordingly, closed expressions for the inverse transformation (joint position as function of target position) are intractable, except perhaps by MATHLAB standards.

This does not necessarily mean, however, that the computation is inherently expensive, slow, or very hard to program. We discuss briefly how it can be done efficiently, using a rather substantial amount of memory but otherwise economically. Then we shall discuss how the principle might be used also as a basis for a theory of more complicated postural control problems.

I have decided not to write this section now, because I don't understand it well enough yet, and waiting for it would delay the rest of the memo too long. Papert and I have examined various special cases, and so have a number of students. The basic idea is this: one divides the real-world space into domains by selecting vertex points for a convex subdivision. The input coordinates (in joint-position space) for each vertex are stored in memory. Then, to find the joint-position coordinates for a point specified by its real-world coordinates, we find the vertices of the convex cell that contains this point, and apply an interpolation formula to the joint-coordinates associated with those vertices. Thus we have a piecewise linear approximation system.

For the human arm, we think that less than 100 points would be necessary to build an adequate partition, within which the interpolation accuracy would be acceptable. The thesis of Gresser, on controlling a redundant (12-degree-of-freedom) arm is relevant to this idea.

Problems to be discussed:

- How does one combine approximations for body, head, eye motions?
- What is a good space-lattice? How many points?
- How does one find the smallest cell containing the goal point?
- What is good interpolation formula?
- What about critical points? How does one treat "dead center" regions. What about global sectioning of general access zones?
- Need the near-points form a convex container? Should data contain derivatives? Should local motions be referred to local coordinate system (resolution into orthogonal components)?
- What about redundant degree systems? Should alternatives be explicit? How about path-planning? Proposal: a small collection of prototype paths, plus perturbation programs.
- How is the table formed? Does one precalculate mathematically? Or can it be learned easily? For the full posture model, there is a problem of staging --- as in finding coefficients of multilevel perceptron.

4. Finger design. The "Escalator Grip"

People are always arguing about how complicated a hand should be, but agree that what we really need is a rational way to discuss such questions.

I wish I had something useful to say about this. In the meantime, we can agree that the issue can be put aside for a time if we make the hand capable of using special-purpose tools. This suggests that as a minimum, we should at least supplement simple gripping functions with a capability for easy power and sensory connections to tools.

Here is an idea so simple that it might be considered a cheat. We provide the inner surfaces of one or two finger-tips with a roller or moving belt that has a high-friction surface. It can be actuated either direction by motor. If the two grippers are slightly divergent, then one can just press the tips against the object to be grasped, and the traction will draw the object firmly into the grip. Delicate rolling and shearing motions are available. Although tactile sensing would be difficult on such surfaces, one could compensate by force-instrumenting the belt supports.

The rollers would serve well as power sources for small power-tools that anchor against the base of the gripper; a two finger grip would supply two independent power feeds into the tool. One could even get three degrees of control, by using the grip motion as well, into a suitably designed micro-tool. For feedback, we might use a fibre-optic coupling.

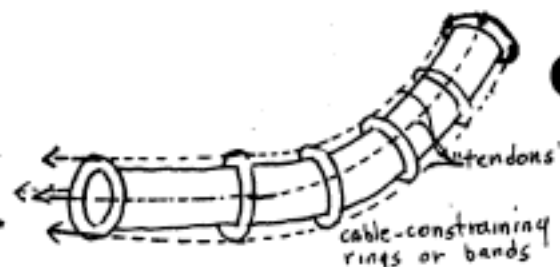
5. The Fourier Wrist Principle

A wrist needs mobility, strength, and a protected instrumentation channel for its hand. To give the hand access to the work from different directions, a wrist should be slender -- the arm should subtend a small space angle as seen from the hand. Because the wrist is near the hand, large angular errors mean relatively small absolute errors, so absolute accuracy of angular control is not very critical. For the systems we envision, wrist control would be adequate even with increments of several angular degrees.

These requirements suggest the use of a deformable tubular structure. In very small scale systems the requirements of strength and rigidity virtually dictate using "exoskeletal" or tubular structures: central rods are not stiff enough and pin-joints are too weak. The tubular construction is also attractive for instrumentation, because the information channels are insulated by the exoskeleton, and the interior is a nearly constant-length environment.

Discrete (localized) joints cause serious problems both for exo- and endo- skeletal structures. The bending radius is very small at a discrete joint, and to avoid this one has to route tendons and other elements along complicated paths.

An apparent solution to all these problems is to use a continuously flexible tube. This section is about some aspects of such a design. The simplest such wrist, no doubt, is a plastic tube or rod deflected by external tendons constrained to its exterior, as shown here. This has problems and advantages:



STRENGTH: Tubing has the best strength-weight characteristics. However, flexible tubing has to be weaker, and tends to buckle and collapse under load. A compromise is discussed below in 5.1.

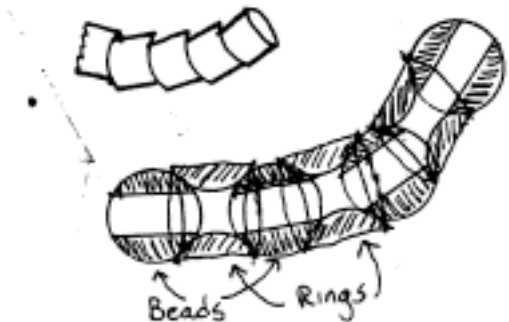
BENDING: Under a pure bending force, the curvature tends to be uniform. Under more complicated loads an elastic beam tends to be wobbly, and bends non-uniformly. We discuss stabilization schemes in 5.2 below.

CONSTRAINT: Tendons tend to span across concave arcs, and have to be constrained. The constraints may cause friction and wear problems.

5.1 "Vertebral" Segmented Columns

Many of these problems stem from the fact that to make a beam flexible we must either make the tubing thin, or use a material with small elastic modulus -- in short, make it weak. One kind of solution is to use a strong material and get the flexibility by slipping.

Two ways to do this are shown here: The segmental method solves the problems of longitudinal strength, tubular collapse, and suggests several methods for tendon constraint. The segmental approach makes the column stability problem worse. It offers the possibility of variable stiffness in exchange: one can use friction to lock the system by pulling all the tendons, to get greater rigidity.



Also, one can make the friction very low, to get an extremely compliant wrist, by relaxing the tendons that act counter to the desired direction of motion. This principle is used in several commercial holding and positioning devices.

An idea that has come up from time to time is to mechanize a vertebral column digitally, that is, to make each joint have just two or four states. One might binary-code the angular deflections. This ought to be feasible, for a wrist, since one ought not need more than about 5 bits of control. The purely binary scheme does not seem too good, however, because one would not want any one joint to have to rotate through a large angle.

5.2 Flexibility and Harmonic Analysis

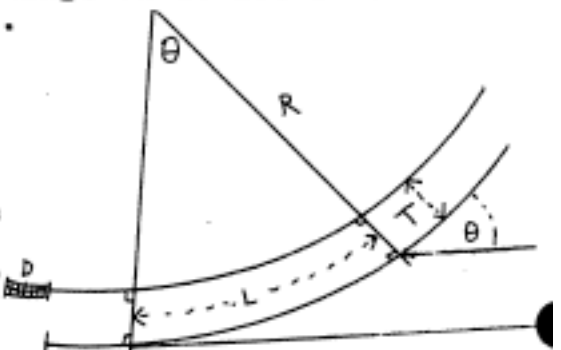
If we want to distribute angular deflections over appreciable path lengths, we have a theoretical problem about controlling the excess degrees of freedom. (To count degrees of freedom for a continuous rod, one has to use a signal theory method. If one describes the configuration in terms of spatial frequencies, there is a rapid attenuation of high frequencies.) In any case, we are interested in the case where there are more degrees than we are prepared to control explicitly.

The curvature in a beam tends to be constant, other things being equal, because the strain energy is a faster than linear function of local curvature. So the energy of the beam has its minimum for a given total curvature when it is uniformly distributed. Unfortunately, this quadratic minimum means that the resistance of the system to perturbations that transfer a small amount of curvature between two different segments is very small. Thus the uniformizing tendency is poorly coupled to the gross spatial configuration.

Now observe an interesting phenomenon that occurs in a flexible wrist with tendons constrained close to the surface of the tube.

To a very good approximation the net change in direction is independent of the tube's conformation.

The basic phenomenon stems from the fact that for small angles the $x = \sin x$ approximation is very good. Consider a flexible rod with opposed deflection tendons, and shorten the upper one a distance D : (Actually, the tendon on the convex side will be pulled in by about the same amount. In the analysis below, let D be the difference.)

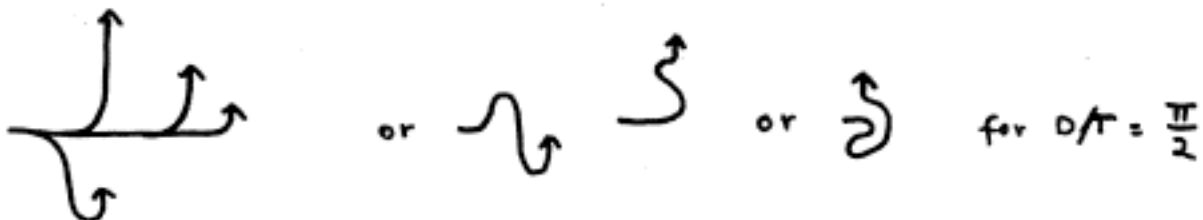


If the rod has thickness T and length L , and one side is D shorter than the other, the rod will bend in a curve for which

$$A(R+T) - AR = AT = D$$

Hence $A = D/T$, where A is the bending angle in radians. Note that this is independent of L .

Now, we assumed uniform curvature in this argument. But suppose that the rod is actually bent sharply in one part and gently in another. Then $A = D_1/T + D_2/T = D/T$ again, so that for a given total difference in tendon length one gets the same net angular deflection of the tip independent of how the curvature is actually distributed along the rod! In other words, the same D is compatible with all configurations that begin and end with the same directions, such as in

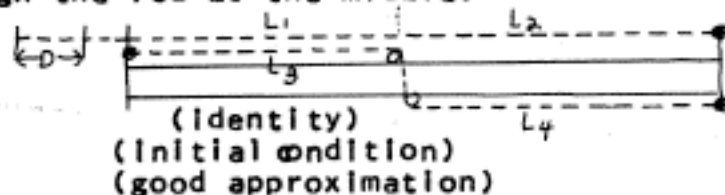


One might be able to make a nice parallel-ruler device exploiting this principle. Handling such a beam is an experience; it seems completely free for translation but firmly constrained for direction of the tip, which seems to have a mind of its own.

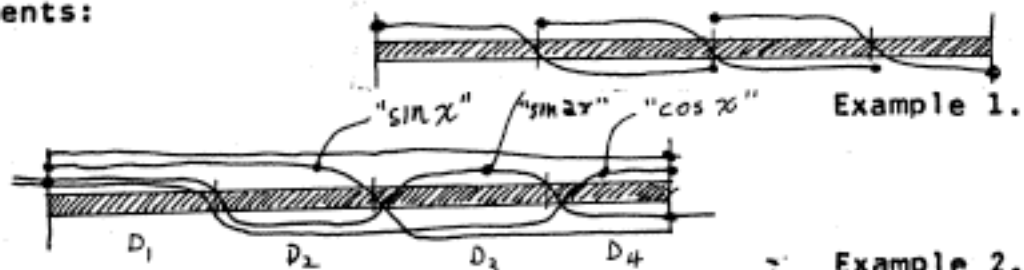
Now to enforce uniform curvature, we must distribute increments of D uniformly (in length) along our beam. Consider a first approximation in which $D = D_1 + D_2$, in two equal halves of L ; then we want somehow to constrain $D_1 = D_2$. Solution: add an extra tendon firmly attached at both ends but crossing through the rod at the middle.

Then one can write:

$$\begin{aligned} L_1 &= L_3 \\ L_3 + L_4 &= L \\ L_2 + L_4 &= L \end{aligned}$$



Hence $L_1 = L_2$. Thus the extra constraint divides the curvature equally between the two halves. One could add still other constraint-cords, crossing over at other points: the problem is to equalize the deflections in each segment. Two examples of how one might do this for more segments:



Discussion of Example 1: To the extent that the oblique cords are inextensible, this scheme ought to be quite effective in uniformizing the deflection. It is like the well-known "lazy-tongs" device. It tends to have a cumulative error, in that the curvature can slowly drift from one end to the other if there are many segments. As drawn, there is also a problem about unconstrained parts of the rod near the cross-over points; this should be corrected by running each constraint over three segments, and this also relieves the sharp curves experienced by the stabilizing tendons.



Example 1'.

Note that each wire needs a symmetrically opposite one for negative curvature control.

Discussion of Example 2: We have shown several constraint cords with suggestive labelling. Each cord runs the full length of the rod, so that the effects are global (no accumulated error). Let D_i and $-D_i$ be the length changes at the top and bottom of the i -th segment. Then we have:

$$\begin{array}{l} \sin A: \quad D_1 + D_2 - D_3 - D_4 = 0 \\ \cos A: \quad D_1 - D_2 - D_3 + D_4 = 0 \\ \sin 2A: \quad D_1 - D_2 + D_3 - D_4 = 0 \end{array}$$

$$\begin{array}{ll} \text{The first two equations yield} & D_1 - D_3 = 0 \\ \text{The last two equations yield} & D_1 - D_2 = 0 \\ \text{The first and last yield} & D_1 - D_4 = 0 \end{array}$$

So all four segments have equal change: the total change D is controlled by the top straight fibre.

Our choice of trigonometric labels was based on a strong analogy. We can use the "constraint" cables for control as well! If we shorten the "sin X " cable by an amount D , the equations still yield $D_1 = D_2$ and $D_3 = D_4$, and (assuming the straight fibre is unchanged) the rod assumes a sigmoid for still with the original tip- direction:

If we let $C(x)$ be the curvature of the rod as a function of length, we now have

$$C(x) = D \sin\left(\frac{2\pi x}{L}\right)$$



Controlling the other fibres similarly, we can develop the curvature of the rod in a Fourier series:

$$C(x) = \sum_j D_{\sin jx} \sin \frac{2\pi jx}{L} + \sum_j D_{\cos jx} \cos \frac{2\pi jx}{L}$$

It would be impractical to do this for more than eight segments, and four would be quite enough for most purposes.

5.3 Three Dimensions

Extending this idea to controlling a space-curve promises to eliminate two nuisances. First, by winding the tendons around the beam instead of through it, we avoid penetrating the central core. (2) This same scheme can eliminate the occurrence of "dead" intervals at which the tendons are near the axis and so provide little constraining force (poor mechanical advantage). Our scheme is to use Helical winding instead of Sinusoidal.

Using helical winding, one avoids the problem of tendons escaping from concave portions of the beam. It appears that a large number of fibres are required.

The four pairs of the 2-d case must each be duplicated in two planes. Our idea is to make two sets of windings, 90 degrees out of phase, in both directions around the cylindrical rod. Each must be duplicated again, in the opposite direction, to oppose the effects of axial torque. That makes 16 pairs of fibres. Since we really do have 8 degrees of freedom, the system certainly needs 8 tendon pairs. Possibly, half the windings could be eliminated if the tube has enough inherent axial stiffness, but this is not easily compatible with longitudinal flexibility. One way to get the required kind of stiffness is to have hinge joints in alternating orientation, as in the lobster arm.

These issues are not so serious. Even a two "segment" arm, in three dimensions, would already be a splendidly mobile device, and its helical geometry problem does not look very difficult. Going to four "segments" may raise more serious problems about friction and tendon-crossovers. Higher terms in the series gets us involved with long cables (the "high frequency" sinusoids wrap around the rod a lot) and would probably cause severe frictional binding problems because the forces normal to the tube would become large. These forces don't look serious for the two and four segment designs.

Problem: Design a three-segment model. There is a problem in the algebra.

5.4 Master-Slave Control

Our "harmonic" wrist lends itself nicely to master-slave control because two of them connected together are constrained to behave identically if we simply cross-over the tendon-pairs at the junction.