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**THE RELATION BETWEEN PROXIMITY AND BRIGHTNESS SIMILARITY
IN DOT PATTERNS**

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ABSTRACT. The Gestalt studies demonstrated the tendency to visually organize dots on the basis of similarity, proximity, and global properties such as closure, good continuation, and symmetry. The particular organization imposed on a collection of dots is thus determined by many factors, some local, some global. We discuss computational reasons for expecting the initial stages of grouping to be achieved by processes with purely local support. In the case of dot patterns, the expectation is that neighboring dots are grouped on the basis of proximity and similarity of contrast, by processes that are independent of the overall organization and the various global factors. We describe experiments that suggest a purely local relationship between proximity and brightness similarity in perceptual grouping.

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1. Introduction

The notion of grouping was introduced by Wertheimer [1923] to describe visual processes that take a collection of discrete elements, such as dots, and produce a more global one, such as a line. His argument was posed primarily in terms of visual demonstrations like the one in figure 1a. These demonstrations were very convincing, and culminated in a series of principles postulated to underlie visual organization (see also [Kochler 1929; Koffka 1935]). Dot patterns have been especially effective in demonstrating grouping by proximity and by similarity. Figure 1a demonstrates the proximity principle: the vertical spacing between the dots is less than the horizontal spacing, hence the dots appear grouped into vertical lines. Similarity among (or between) dots also influences the perceived organization: if the dot spacings are equal in the horizontal and vertical directions but the dots in alternating rows have similar intensity, one sees horizontal lines. Since the proximity and similarity principles are independent, they can also be brought into "conflict", with proximity suggesting one organization (say, horizontal) and similarity another.

The rectangular grid patterns used to demonstrate these principles are highly regular and symmetric, of course. The tendency to perceive organizations that are regular or symmetric, two global properties of a pattern, was also described by the Gestalts as a principle or law of organization. Do all these factors, proximity, similarity, regularity, symmetry, and others, simultaneously combine within a single perceptual process, or is there a succession of processes: local processes that impose groupings, say, between dot pairs, and subsequent processes that build more global organizations out of the local groupings? The end result in either case is an organization among dots that is influenced by many factors; the difference lies in how the factors are applied and how the organization is constructed. There are several reasons, which we will discuss, for expecting multiple processes. The main purpose of this article is to demonstrate that the influence of proximity and similarity (of dot contrast relative to background) on dot grouping is substantially independent of the global organization that emerges, which supports the conjecture of initial local groupings.

Three types of dot pattern were examined, each consisting of two populations of dots of differing contrast relative to the uniform background. The first is a rectangular grid that appears organized into columns when the dots are of similar contrast (figure 1a) or rows when they are of very different contrast (figure 1b, where one population has zero contrast relative to the background to emphasize the alternative organization). The second pattern is an interesting variant that, depending on the contrasts of the two populations of dots, yields two very different apparent organizations: a horizontal/vertical grid or a diagonal grid (figure 1c). The third pattern is a circular "Glass pattern" (figure 1d), created by displaying a pattern of random dots, and then superimposing a copy of these random dots rotated by a small amount (see [Glass 1969; 1979]). These patterns, despite their different global structure, exhibit very similar quantitative behavior in terms of contrast and proximity. The psychophysical observations reported here suggest that the organization may be accomplished by the same perceptual process (or several that are similar). Furthermore, this process computes a local quantity, which we shall view as (an approximation to) the tangent to the perceived grouping contour at each dot, and which we expect to be locally computable.

As background we discuss, in the next section, the analogous relationship between the aggregation of dots into contours and the problems of contour aggregation in general. The similarities suggest that dot aggregation may involve some of the aggregation processes that generate continuous contours from local edge

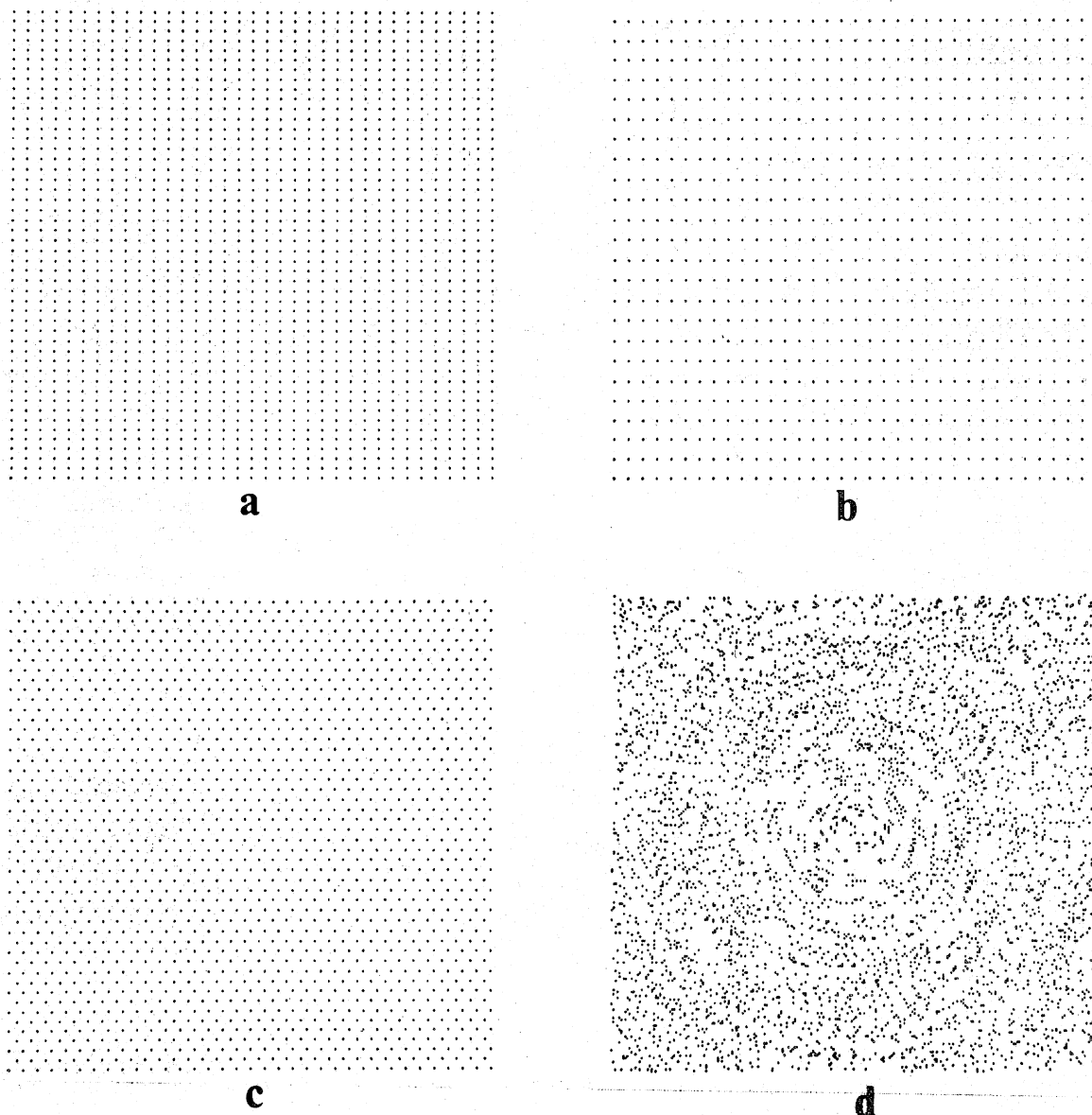


Figure 1. The dot patterns used to examine the role of contrast similarity and proximity in one-dimensional grouping. *a* shows a vertical organization, *b* a horizontal one, *c* a "hash" organization, and *d* is a circular Glass pattern (see text for description).

or line information. The initial aggregation processes (particularly those that impose local groupings between dots) probably differ, but the later processes are probably similar. Briefly, we will review evidence that dot contours are not merely detected by "line detector" mechanisms such as simple cells, but instead, by processes that build from very local groupings between neighboring and similar dots. The global dot contour then emerges by processes that aggregate together the local dot groupings. The first stages of contour aggregation, therefore, are expected to be purely local. Following this discussion we present supporting experimental evidence.

2. Theoretical discussion

Why does the visual system, when presented a set of discrete dots, tend to organize into contours those dots

that are adjacent, similar and collinear? The answer to this question comes, we believe, from an examination of the role that continuous intensity contours play in vision -- they signal physical discontinuities of various kinds, such as illumination or reflectance discontinuities, physical creases or surface boundaries. The intensity discontinuity on the retina is detected by collections of cortical neurones organized into local operators such as those modelled by Marr & Hildreth [1980]. This detection process, however, does not immediately yield a globally contiguous contour, for a variety of reasons. The local operator responses are confounded by several factors, e.g., occlusions of objects by foreground details in the scene, insufficient contrast in places, and various transducer artifacts of early visual processing. Consequently, rather than producing responses along a continuous contour path, they result in a pattern of contiguous fragments and occasional gaps (see [Marr & Hildreth 1980] figure 8d). Some additional processing is clearly necessary for completing these fragments; i.e., for grouping them into a complete contour. Gestalt similarity notions are seen to be related when one considers that the contour fragments arise from the images of smooth physical objects. That is, pieces of a physical contour project to fragments which are generally similar in contrast, intensity, motion (if any), and furthermore are proximate and roughly collinear. These properties are used as local evidence for reconstructing the whole contour from its discontinuous fragments [Marr 1976, 1982; Zucker 1981]. (Note that a form of this argument would hold for any local contour-detection operator.)

Dot patterns provide a particularly useful tool for studying contours because the items themselves are discrete but the contours appear continuous. They provide an example of contour completion that is especially simple, because the discrete segments, the dots, have no intrinsic orientation. Thus empirical observations on dot pattern grouping may be seen as a first step toward understanding groupings between more general (and, in particular, oriented) visual entities.

Psychophysical research on dot grouping has concentrated primarily on two-dimensional texture discrimination, wherein a dot pattern segregates into regions delimited by boundaries across which certain statistics vary discontinuously [Julesz 1962; Beck 1966; Olson & Attneave 1970; Julesz 1971]. Subjectively, there is cohesion among similar texture elements and segregation between dissimilar elements. A common expectation is that the perceptual segmentation of texture arises by two-dimensional aggregation of similar texture elements; the similar elements cohere into demarcated regions. From this point of view it is conceivable that the aggregation of dots into one-dimensional chains is but a limited case of more general similarity grouping, particularly in view of the comparable role of brightness and proximity in each case (see [Rock & Brosnole 1964; Hochberg & Silverstein 1956; Prytulak & Brodie 1975]). But one cannot conclude from this analogy between one- and two-dimensional aggregation that the empirical results regarding the two-dimensional task apply equally to the one-dimensional. In fact, the computational task of texture discrimination (regardless of whether one views it as segregation on the basis of dissimilarity or aggregation on the basis of similarity) is likely very different from that underlying the aggregation of discrete items into continuous contours. Texture discrimination probably pertains to the localization of physical surface boundaries, while the one-dimensional aggregation pertains to localizing line-like contours. (see [Zucker 1982] for more discussion). Moreover, dot contours exhibit several distinct perceptual differences from texture boundaries (e.g. regions differing in dot density) [Stevens 1982]. Thus while texture boundaries and line-like contours exhibit the various Gestalt grouping principles, important differences are revealed on closer

examination.

The locally-detected evidence for a contour can be modelled, in the case of a continuous locus of retinal intensity change, as a bar or edge assertion. A bar or edge corresponds roughly to the tangent to the contour at a point.¹ In the analogous case of a dot contour, a chain of collinear and similar dots within a dot pattern, the local evidence can again be modelled as the tangent to a dot contour. One particular example is a pairwise association between neighboring and similar dots, which Marr [1976] calls a virtual line; for further examples see [Zucker 1982]. As is expected with bar or edge assertions, the tangent to the curve is expected to be computed by a process having local support.²

We expect, therefore, that the local groupings in dot patterns are achieved by processes that make the tangent orientation explicit. Moreover, the relationship between local dot groupings and an overall dot contour is viewed as similar to that between edge or bar assertions and a continuous intensity contour. In particular, the local groupings are computed on the basis of purely local evidence, including the proximity and similarity of dots.

3. Experiments

The purpose of the experiments was to determine the relationship between proximity and similarity (of intensity) in the grouping of dots for various dot patterns. The basic experimental paradigm was an extension of that implicit in the Gestalt demonstrations. Subjects viewed brief displays and were forced to either state the perceived grouping, or to state that none was present. Each experimental pattern was designed to have two very different apparent organizations, such as the vertical versus horizontal in figure 1a, which allowed a simple forced-choice protocol. Our objective was to quantify the strength of the perceptual grouping into one of the two apparent dot organizations as the similarity of neighbouring dots was systematically varied. Spacing and intensity (of both the dots and the background) were varied.

3.1 Method

3.1.1 Subjects

There were eight subjects, five males and three females. Two had experience with the experimental tasks and were aware of the goals; the rest were naive in both regards. All had normal or corrected vision.

1. Plus information about contrast, motion, and so forth -- see Marr's [1976] discussion of the elements of the primal sketch. See also [Zucker 1982] for discussion of the relationship between simple cell responses and the tangent to a curve.

2. A dotted line, or a simple pair of dots, may approximate a continuous line segment in the relatively low spatial frequencies (a blurred dotted line resembles a blurred continuous line). In this connection simple cells [Hubel & Weisel 1962] have been proposed as responsible for detecting pairs of dots in Glass patterns (such as figure 1d) [Glass 1969; Glass & Switkes 1976; Glass 1979], and similarly for detecting triples or longer chains of roughly collinear dots [Caelli & Julesz 1978]. But it should be noted that the tangent to a dotted line is probably not directly provided by simple cell-type receptive fields applied to the intensity distribution. Stevens [1978] and Carlson *et al.* [1980] have devised various dot patterns whose perception are not easily explained by simple cell-like mechanisms applied directly to the intensity distributions, and careful examination of the low spatial frequency content of dot patterns shows that detecting a dotted line by, in essence, its blurred image would seldom be successful [Stevens 1982]. Thus while cortical units such as simple cells might contribute to the detection of closely spaced dots, that cannot be the sole mechanism involved in dot contour aggregation.

3.1.2 Apparatus

The stimulus images were generated by a Grinnell GMR-27 graphic display processor on a DEC VAX 11/780 computer and displayed on a Tektronix 670A-1 color monitor. The experiment was conducted in a dimly illuminated room, with a background reflectance of 0.58 candelas/meter² from the monitor. The subject sat on a chair two meters from the monitor and, while instructed to sit still, was permitted slight movement.

3.1.3 Procedure

The experiments consisted of several sessions. During each session, the S was presented with a sequence of images which were constant in dot proximity but variable in similarity. Each image was flashed for 250 msec. The Ss viewed a uniform grey display between stimulus flashes, of 133 candelas/meter², chosen to be intermediate between the calibrated minimum and maximum intensities. Post masking (for instance by a random dot pattern) was considered unnecessary for this task. For each stimulus presentation, the S responded with one of three answers, e.g., for the patterns of Figure 1a the choice was between "horizontal" and "vertical", depending on perceived linear orientation, and "nil" when no dots could be distinguished (this occurred when they were all at one intensity with the background). The S responses were recorded and plotted as the accompanying graphs.

To become familiar with the task protocol each S was given a trial run of ten images before beginning the data collection. All Ss used in the experiment grasped the procedure within these first ten trials.

3.1.4 Stimuli

Three types of dot pattern were used as stimuli: *i*) those with a horizontal or vertical orientation (figure 1a and b); *ii*) those organized in either a horizontal/vertical direction or a diagonal direction (which we refer to as "hash" patterns, figure 1c); and *iii*) rotated Glass patterns (figure 1d). Each type of pattern consisted of two overlapping populations of dots, one was fixed in intensity while the other varied. All dots subtended 1.2' of arc. Two apparent organizations could be seen depending on the similarity in dot intensity between the two populations (discussed in more detail momentarily). There were 25 different images of each type, constructed from variations in the intensity of background and variable-intensity dots. For each presentation, the variable set of dots and the background independently took on one of five intensity values between the minimum and maximum calibrated intensities of the display monitor. The fixed-intensity dots were set to the maximum intensity. The S was shown a sequence of 125 images (each of the 25 combinations of variable-dot and background intensities was presented a total of five times), in random sequence. The stimulus patterns will now be described in more detail.

Horizontal/vertical

A typical image from the horizontal/vertical discrimination experiments is given in figure 1a; it is also presented schematically in figure 2a. Here, the x's represent dots with intensity fixed at a maximum over all images of a sequence, and the o's represent dots with variable intensity. The intensity of the background was also variable from image to image. The distance between the horizontal x's was 10' of visual angle; the distance between vertical x's was 13'; and the o's were intermediate between the vertical x's. Thus, with all dots at similar intensity and substantially above background the pattern appears vertical, as in figure 1a; with

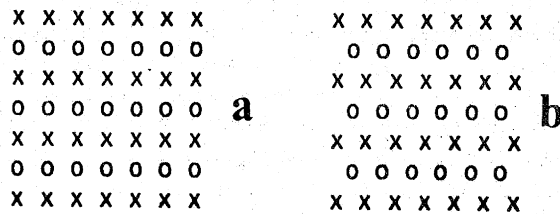


Figure 2. Schematic diagram where x's are dots of maximum intensity, o's are dots of variable intensity.

the o's at background intensity the pattern appears horizontal as in figure 1b.

The three responses for these images were "horizontal", "vertical", and "nil", depending on the predominant orientation (if any) seen in the pattern. To compensate for any bias toward horizontal or vertical, the patterns were presented as in the above figure or rotated by 90° (each orientation had probability 0.5). The same patterns were presented in later experiments where all dot spacings were doubled.

Linear/hash

The linear/hash figures are similar to the vertical/horizontal. An example is given in figure 1c, and shown schematically in figure 2b where the x's represent dots at fixed maximum intensity, and o's are dots at variable intensity. The spacing between the horizontal x's was $10'$, and $13'$ between the vertical x's. The o's were intermediate between the x's in both directions, giving a spacing between x's and o's of $8.2'$.

The choice of responses for these patterns was between "linear" (when either horizontal or vertical organization was seen), "hash" (when a grid-like or diagonal organization was seen), and "nil" (when no dots were seen).

Half the images for each type of pattern were presented as above and half were rotated by 90° . The same patterns were presented later at twice the scale.

Glass patterns

The images presented in these experiments were rotational Glass patterns, similar to that in figure 1d. They were created by rotating a pattern of 1000 randomly generated dots by 5° and displaying the original and transformed dot patterns together. The original dots were at a fixed intensity over all the images of a stimulus sequence, and the intensities of both the transformed dots and the background were variable. The responses were chosen from among "grouped" (for a circular organization), "ungrouped", and "nil".

4. Results

The variable-intensity dots are considered to have "interfered" with the perception of the pattern of fixed-intensity dots when the organization of that pattern is perceived as changed. That is, the variable dots interfere when they group with the fixed dots and change the apparent organization. One organization is given by the o's in our schematic diagrams; the other is given by the x's and the o's taken together. For example, in the horizontal/vertical figures the linear organization may be horizontal or vertical, depending on the intensity relation between the two sets of dots and the background. Interference, or grouping, in the

linear/hash pattern produces a hash-like organization; in the rotational Glass patterns it results in circular organization.

Our results are shown as graphs, in which the interference of the variable-intensity dots is plotted as a function of the intensities of the background and of the variable-intensity dots. All graphs are to the same scale, with the maximum indicating that five interferences were reported in the five presentations of that combination of variable-dot and background intensities. (In this case the fixed and the variable dots would have grouped completely.)

Figure 3a shows a typical result for one S in the vertical/horizontal task. For this experiment, the fixed dots were at maximum screen intensity. The graph can be read as follows. For each value of the background, a curve is specified that shows the intensity of the variable dots that grouped. Running in the opposite direction are curves corresponding to fixed values for the variable dots; they show the variation with changes in the background level. Taken together, the two families of curves specify interference grouping as a function of both of these variables. The presence of the peaks completely on one side of the diagonal indicates that the S never grouped dots with opposite contrast. As the background became lighter, the variable dots had to be lighter before they interfered and defined a new organization. Note furthermore that, for this intensity quantization, about one step difference from the background was sufficient for interference, with a bit more for the lower values of intensity.

This result is typical of those obtained for all three grouping tasks. In figure 3b we show the mean for 6 S's, with the standard deviation in figure 3c, for the vertical/horizontal task. In figure 4a we present a typical result from the linear/hash task, with the mean and standard deviations for 4 S's in figure 4b and c. This task showed more variation than the vertical/horizontal, and the Ss found it harder to decide between the two organizations (there being a wider range over which either organization could be seen). The third task, that involving Glass patterns, was the easiest to perform, and shows the "cleanest" results; see figure 5a for a typical response and figures 5b and c for statistics over 3 representative Ss. In each case dots with similar

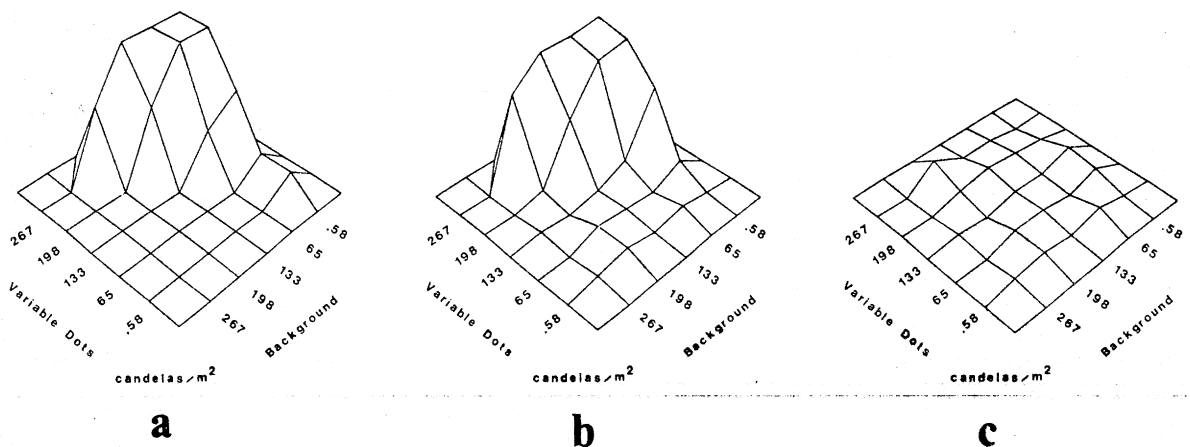


Figure 3. Results of the vertical/horizontal task. *a* shows a typical result for one S. The two axes denote intensity scales for the variable dots and for the background. Height is proportional to the percentage of interferences, or groupings, of five trials for each combination of background and variable dot intensity. *b* shows the mean performance and *c* the standard deviation over six S's.

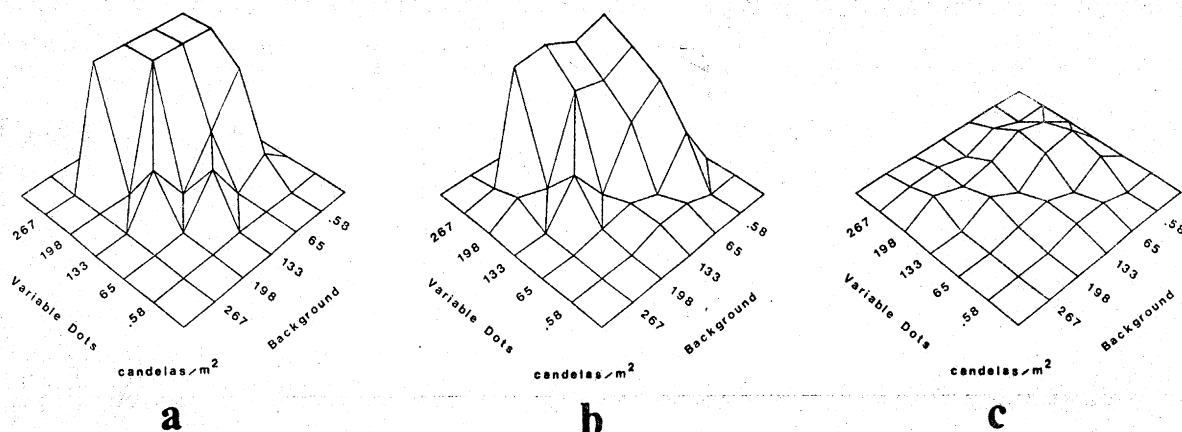


Figure 4. Results of the linear/hash task. *a* shows the result for one S, *b* the mean and *c* the standard deviation for four S's. Note that this task was much less clear than either the vertical/horizontal or the Glass pattern task.

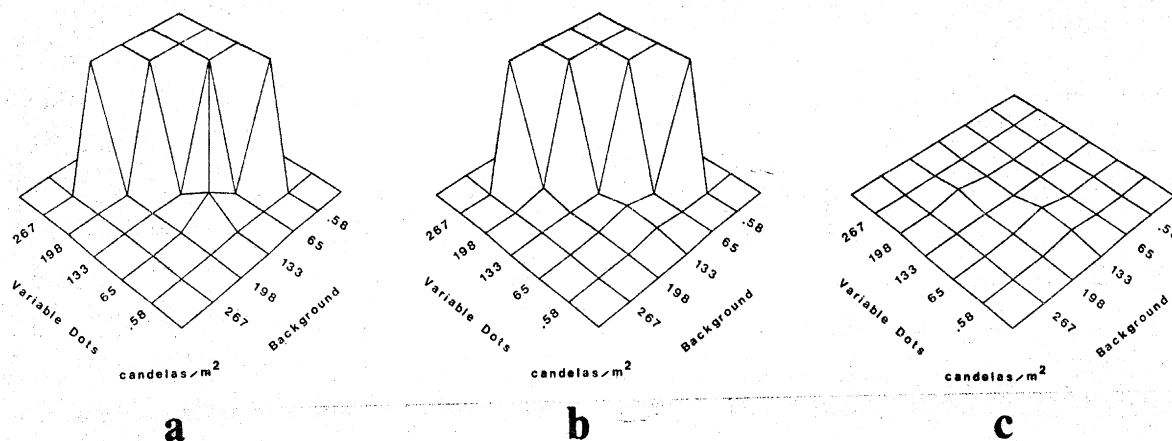


Figure 5. Results of the Glass pattern task. *a* result for one S, *b* the mean and *c* the standard deviation for three S's. The sharp "corners" in the above graphs indicate that this task was the easiest to perform; as long as the variable dots were of the same contrast as the fixed dots, the groupings formed.

contrast (relative to the background) group together, provided they are readily distinguishable from the background. Note from the graphs that interference requires less intensity difference (between fixed and variable dots) at the higher intensities than at the lower for both the vertical/horizontal and the linear/hash tasks. These findings continued in subsequent experiments.

The next series of experiments were repetitions of the vertical/horizontal task with twice the spacing between dots (20' and 13' rather than 10' and 6.5'). The fixed intensity dots were constant at 267 candelas/meter². The data is qualitatively very similar; see figure 6a and b for the statistics over three S's. It is worth noting that the task is much more difficult to perform at the longer inter-dot spacings. The impression of dotted lines, both vertically and horizontally, is much less vivid, and the task judgment seems to be made on the basis of shorter dotted segments, or sometimes even mere pairings. Doubling the spacings again emphasizes these perceptual differences from the original, closely-spaced task, with the result that, for

two S's, dotted lines were clearly seen only for intensity combinations with the background dark and the variable dots light (within two quantization steps of the fixed, bright dots). But the task could still be performed with results like those in figure 6.

The above experiments were conducted with the fixed dots at the maximum screen intensity. In order to show that the results hold with variations in this intensity, in the next series of experiments they were repeated with it set to an intermediate gray level. Now it becomes possible for the dots to group on both sides of the background, or, in terms of the graphical display, on both sides of the diagonal. The result is a much more complicated display, with the peaks previously confined to one side now present on both. The results for one S performing the Glass pattern task (with the fixed dots at 133 candelas/meter²) are shown in figure 7a; the statistics for 3 S's performing all three tasks at this intensity level are in figure 7b and c. Note that the variance is now a bit higher, but that the curves are very similar. Finally, in figure 8a and b we show the results from one S performing the three tasks with the fixed dots at 0.58 candelas/meter². It is essentially the "mirror" image of the original task results (cf. figure 3). It is especially interesting that, like in the original task, the slope is steep when the variable dots approach the fixed dots in intensity, but is very gradual (over two quantization steps) when they are at opposite ends of the scale.

Since the Glass pattern task provided the least variance, we chose it as the task for a closer examination of the effect of intensity on grouping in the range where the transition occurs. The original experiment was repeated with a much smaller range of intensity variation between the fixed and variable dots. The results for two S's are shown in figure 9a and b. The background was set to 133 candelas/meter², and the fixed dots were at full intensity. The transition is rather abrupt.

In summary of these data, we feel that a real pattern has emerged. It is illustrated in figure 10a and b, which shows the statistics for all three tasks performed by 8 S's (with the fixed dots at maximal intensity and the small spacing between them). Dots virtually never group across the background, and readily group when they differ from it. This observation generalizes one made by Glass & Switkes [1976] regarding Glass patterns containing dots of opposite contrast; see also [Julesz 1965]. The variability occurs when the dots are close to

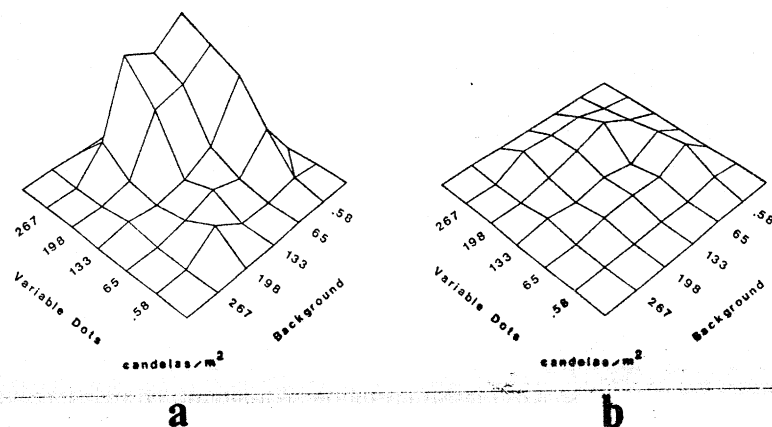


Figure 6. Repeat of the vertical/horizontal task with twice the spacing between dots. *a* shows the mean and *b* the standard deviation for three S's. Note that the results here are much noisier than for the more closely spaced tasks.

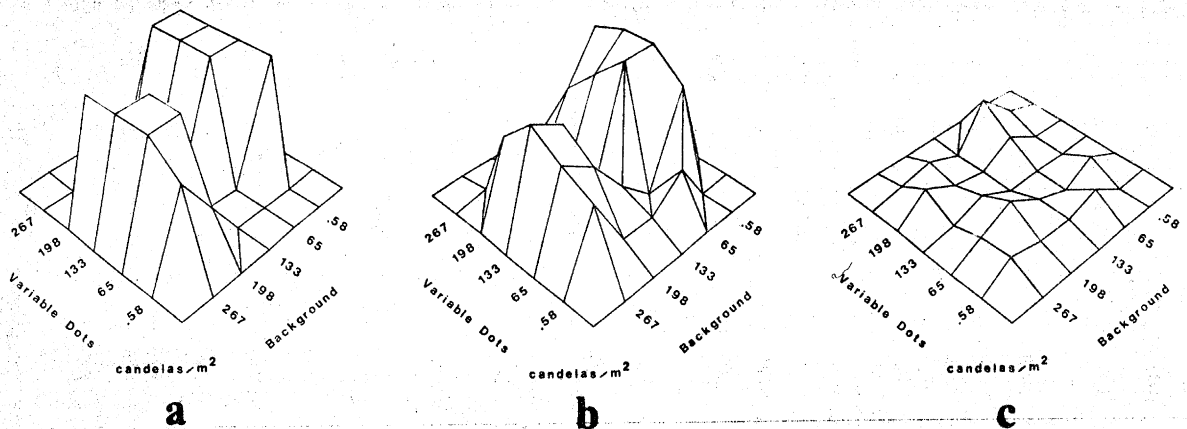


Figure 7. Results of Glass pattern task with background at intermediate grey level, and dots of same or opposite contrast sign relative to background. It now becomes possible to form two separate groupings, one when the dots are brighter than the background, and the other when they are darker. Hence there are two peaks in the graph, one on either side of the diagonal. *a* shows the results for one S, *b* the mean and *c* the standard deviation for three S's.

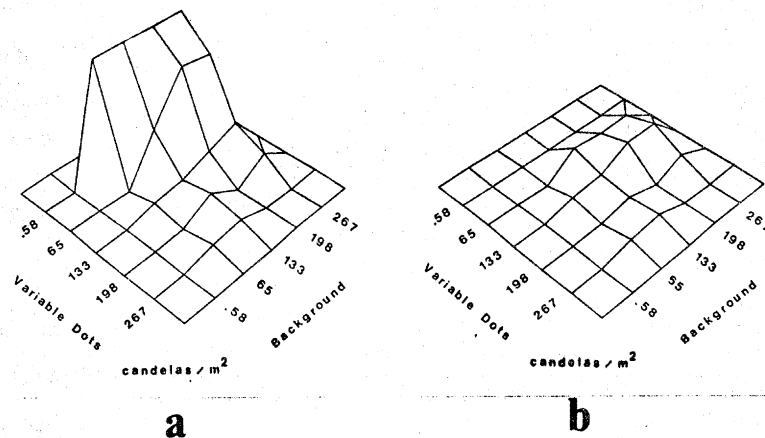


Figure 8. Horizontal/vertical task (figure 3) but with fixed dots at 0.58 candelas/meter². Since the fixed dots are now dark, the result is symmetrically the same as that in figure 3.

the background, as is indicated by the way in which the two "tails" of the peak slope gradually down to zero. It is also shown in the standard deviation plot of figure 10b, which is highest at these two places. But this summary graph obscures the individual task differences discussed previously.

5. Further experiments

All of the tokens in these grouping tasks have been dots, and the question immediately arises as to whether small variations in these tokens would still permit grouping. We attempted two such variations -- the variable dots were made larger, by a factor of two (so that they were no longer pinpoints), and were grown into short line segments by increasing their length by a factor of three. An example of these latter patterns is shown in figure 11. It immediately became clear, however, that when these new images were substituted into our

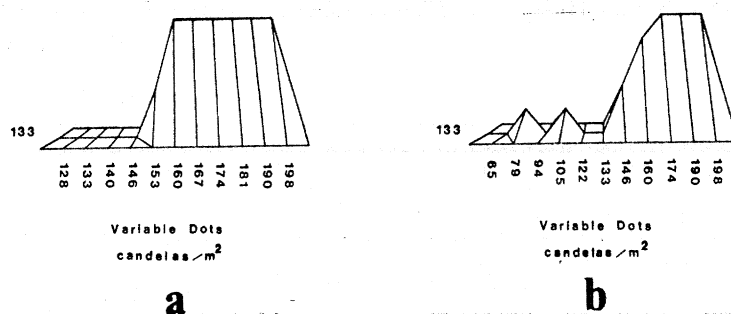


Figure 9. Repeat of Glass pattern task with smaller range of intensity variation between the fixed and the variable dots. Results for two S's are shown. Note that, in both cases, the slope is rather sharp.

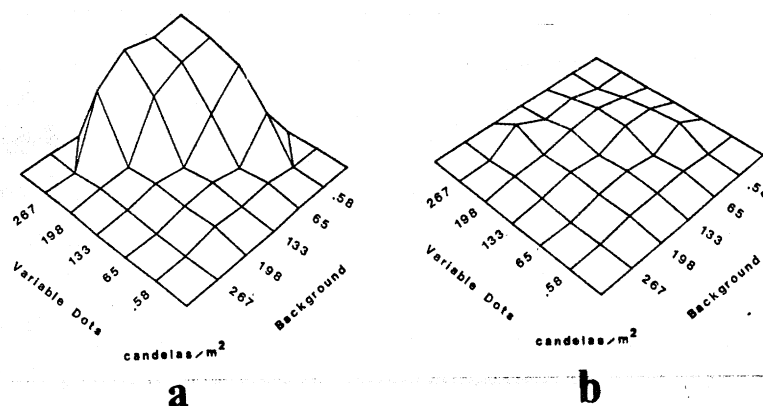


Figure 10. Statistics across all three tasks and subjects, *a* mean and *b* standard deviation.

experimental protocol for the vertical/horizontal or linear/hash tasks, the new tokens always grouped separately from the original dots. Thus the visual system is very sensitive to differences of "type", apparently more so than to small intensity changes, and that these differences predominate over intensity differences for these two tasks. Subjects could "cognitively" report the groupings, and could decide whether or not to group dots with lines, but the immediate perceptual effect that is so clear with dots was gone. Every S reported this difference in the task, and no data were formally collected.

6. Conclusions

Grouping is a generic term for processes that take collections of perceptually distinct visual elements and form them into more global, and perhaps more abstract, wholes. It seems clear that there is a diversity of such processes, ranging from functions very early on in visual information processing, such as completion of intensity-discontinuity contours, to later processes involving surfaces. We argued in this article that the

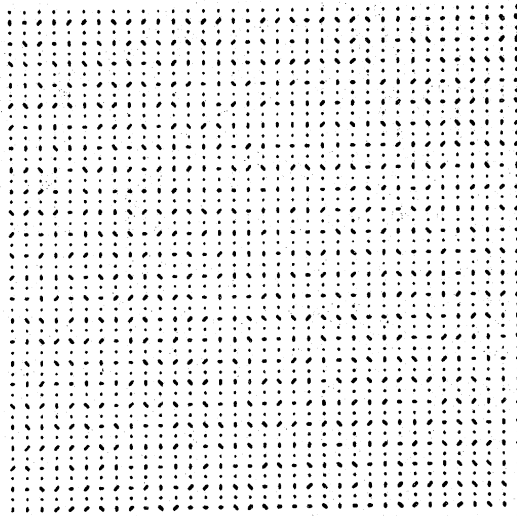


Figure 11. Example pattern combining dots and short line segments.

Gestalt demonstrations involving dots were exercising this former system in a controllable manner. Hence they are amenable to study, and that was the primary purpose of this article.

In particular, we studied the psychophysics of three dot grouping tasks, where spacing, intensity, and shape parameters were varied. Subjects behaved very similarly for these three tasks, in that they always grouped dots that were immediately distinguishable from the background, provided they had similar contrast with respect to that background.

These results suggest two principle conclusions. Firstly, since the psychophysics are very similar across the different tasks, it seems likely that they are resulting from the same kind of (if not the same) process. Secondly, since the global geometry of the contours is different in the various stimulus patterns the processing would seem to be after a local quantity. Both of these conclusions are consistent with theoretical activities in early visual information processing, which were used to argue for the existence of a mechanism for contour completion. Such mechanisms would have both of the above properties.

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