MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ARTIFICIAL INTELLIGENCE LABORATORY

AI Memo 951 *November 1987*

Comparative Analysis

Daniel S. Weld

Abstract

Comparative analysis is the problem of predicting how a system will react to perturbations in its parameters, and why. For example, comparative analysis could be asked to explain why the period of an oscillating spring/block system would increase if the mass of the block were larger. This paper formalizes the problem of comparative analysis and presents a technique, differential qualitative (DQ) analysis, which solves the task, providing explanations suitable for use by design systems, automated diagnosis, intelligent tutoring systems, and explanation based generalization.

DQ analysis uses inference rules to deduce qualitative information about the relative change of system parameters. Multiple perspectives are used to represent relative change values over intervals of time. Differential analysis has been implemented, tested on a dozen examples, and proven sound. Unfortunately, the technique is incomplete; it always terminates, but does not always return an answer.

This paper will appear in *Artificial Intelligence* in July/August 1988.

 This report describes research done at the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology. Support for the laboratory's Artificial Intelligence research is provided in part by the Advanced Research Projects Agency of the Department of Defense under Office of Naval Research contract N00014-85-K-0124. Portions of this research were done at Xerox Palo Alto Research Center and at Schlumberger CAS Palo Alto Research.

Contents

1 Introduction

The problem of symbolic analysis of real-world systems is central to many problems in artificial intelligence. In order to cope with a changing world one must be able to understand its behavior. Recently, considerable emphasis has been put on a specific kind of analysis: qualitative simulation[2,7,26,15,21]. Qualitative simulation seeks to produce a description of the behavior of a system over time, often in the form of a tree of histories of the system's qualitatively interesting changes over time [23].

This paper discusses the problem of comparative analysis, in many ways the complement of qualitative simulation, and describes an implemented, sound solution technique called differential qualitative (DQ) analysis. Whereas qualitative simulation takes a structural description of a system and predicts its behavior, comparative analysis takes as input this behavior and a perturbation and outputs a description of how and why the behavior would change as a result of the pertubation.

For example, given the structural description of a horizontal, frictionless spring/block system (e.g., Hooke's law), a qualitative simulator would say that the block would first move one direction, then stop, then reverse, etc. A description of oscillation would result. Comparative analysis, on the other hand, takes this description of oscillation and evaluates the effects of perturbations. For example, it would deduce that the period would lengthen if the mass of the block were increased, and explain why. Just as qualitative simulation works without explicit equations for the value of each parameter as a function of time, comparative analysis does not need a formula for the period of oscillation.

The importance of the qualitative approach to comparative analysis is the resulting explanation of *why* the behavior changes. If it weren't for the explanation, one might simply solve a differential equation model using using symbolic or numeric techniques. Many artificial intelligence problems, for example design, diagnosis, and intelligent tutoring systems, have comparative analysis as an important component; the explanation is used in many different ways.

- One way method of automated design is the principled modification of previous designs [25]. For example, suppose a library design for a VLSI pullup circuit has too long a rise time. If the problem solver considers increasing the width of some wire to decrease the rise time, it would like to know the ramifications of this modification *relative to the initial behavior*. Will the delay decrease? What happens to power dissipation? Comparative analysis answers these questions, in qualitative terms, as is appropriate for initial design evaluation. By analyzing an explanation for why the changes happen, the problem solver could focus on further changes to counteract undesired effects.

- Many of the programs which perform diagnosis from first principles use similar generate and test paradigms [9]. Comparative analysis can simplify diagnosis of continuous systems (such as analog electronics) in two ways. Comparative analysis provides a direct test for certain hypothesized faults; if one suspects a resistor of a low value, comparative analysis can predict the resulting behavior. If this prediction does not match the observed behavior, the generator might use the explanation to suggest or rule out additional candidate faults.

In addition, the specific type of comparative analysis discussed in this paper, DQ analysis, can be used backwards to generate candidate faults. If an output voltage measures too low, reversing the inference rules of section 3 might lead to the hypothesis that some capacitor has too high a value.

- A key subproblem of intelligent tutoring systems (ITS) is the automatic explanation of the behavior of complex systems. Most AI work in this direction has focused on the role of qualitative simulation when explaining the mechanism through which devices achieve functionality [19,8]. Qualitative simulation is a critical component of explanation generation, but understanding how systems respond to changes is also important. One doesn't really understand the workings of a refrigerator, if one can't explain the effect of a stronger compressor on efficiency and minimum temperature.

The rest of this paper shows how DQ analysis can solve comparative analysis problems and produce clear explanations as well. The trick to DQ analysis is the use of multiple 'perspectives' to define relative change. Inference rules manipulate these relative change values to generate causal arguments that solve comparative analysis questions.

The rest of the introduction explains how differential qualitative analysis solves comparative analysis problems. Section 1.1 presents more detail about the spring/block example[1] to illustrate the important notion of 'perspective'. Section 1.2 introduces a heat exchanger example to emphasize the importance of considering multiple behavioral topologies. Together these two sections show the range of questions that the differential approach to comparative analysis can answer. Section 1.3 suggests a different approach to comparative analysis: a novel technique called exaggeration. Finally, section 1.4 gives an overview of the remainder of the paper.

1.1 Perspectives

Perspectives are the most important concept in DQ analysis; they are best introduced with an example. Consider an ideal spring attached to a block on a frictionless table (figure 1).

[1] All the examples in this paper, and a dozen more, have been implemented and tested.

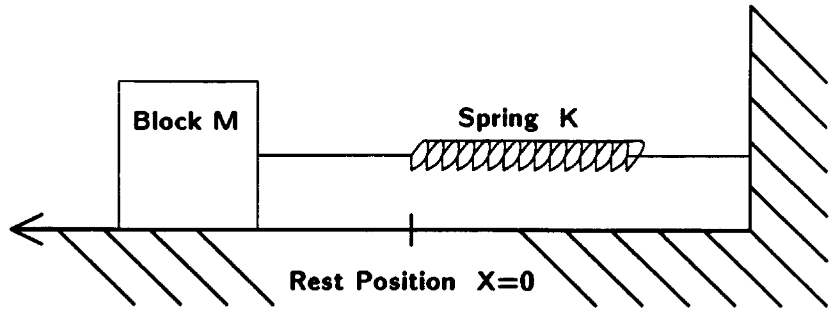

Figure 1: Ideal Spring Attached to Block on Frictionless Table

The system can be defined in Kuipers' QSIM [15] notation in terms of six parameters, each a function of time: spring constant $-K$, mass M, position X, velocity V, acceleration A, and force F related by Newton's second law ($F = MA$) and Hooke's law ($F = -KX$). Mass and spring constant are independent parameters that remain at constant values over time. The initial conditions are specified as follows: $M(0) > 0$, $-K(0) < 0$, $V(0) = 0$, and $X(0) = x_0 < 0$.

This description may now be simulated, but because of ambiguities inherent in qualitative values [14], QSIM produces several possible behaviors for this system, including ones corresponding to increasing, decreasing, and stable oscillation. Although comparative analysis could be done on all of these behaviors, for this example, I assume the interpretation of stable oscillation (figure 2).

Now we are ready to pose a comparative analysis problem.

Example 1 *What happens to the period of oscillation if the mass of the block is increased?*

The answer is that the length of the period increases:

> Since force is inversely proportional to position, the force on the block will remain the same when the mass is increased. But if the block is heavier, then it won't accelerate as fast. And if it doesn't accelerate as fast, then it will always be going slower and so will take longer to complete a full period (assuming it travels the same distance).

What kind of information is needed to produce this explanation? Take the first step: "The force on the block will remain the same." Figure 3 shows a real-valued plot of force versus time. The graph of force in the perturbed system is drawn with a dotted line; in the text I will distinguish the two parameters by calling the perturbed force $\widehat{F}$.

Clearly, $F \neq \widehat{F}$ as a function of time. The corresponding values of F and $\widehat{F}$ are different for almost every possible time. The real meaning of "The force

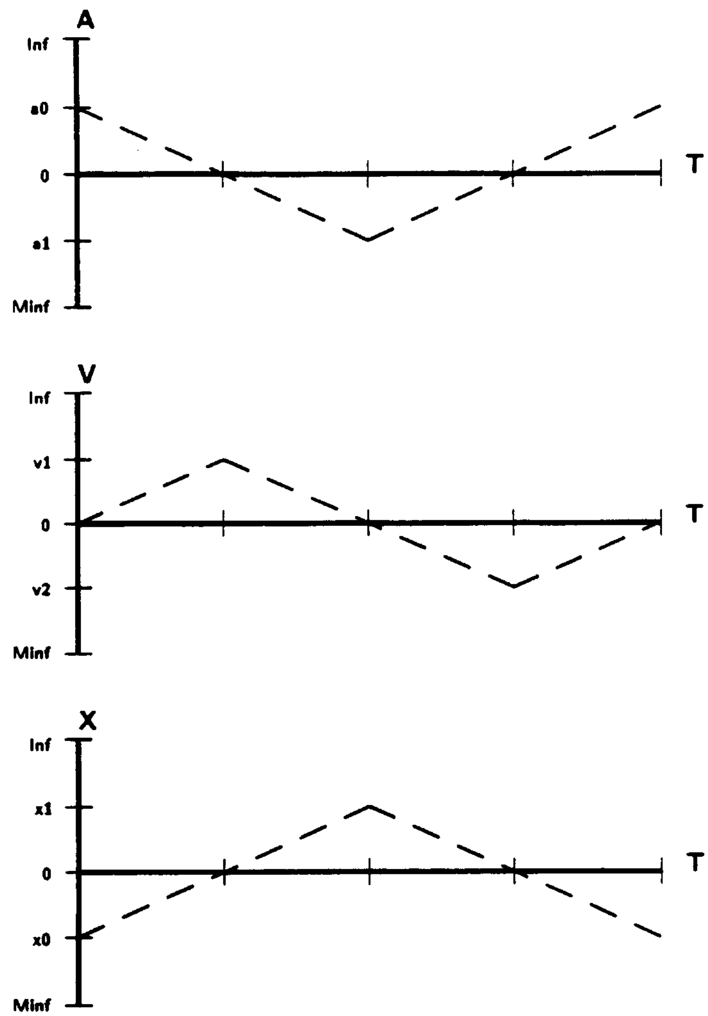

Figure 2: QSIM Behavior for Stable Spring Oscillation

on the block will remain the same" is that F and $\widehat{F}$ are the same for all values of X. Although this reparametrization was not mentioned explicitly in the explanation, it is essential to the soundness of the argument.

In order to allow programs to generate and evaluate explanations like the one for the spring and block, it is necessary to take this implicit concept and make it explicit. I do this with the use of 'perspectives'. Thus the first line of the argument could be rewritten "If the mass is increased, force does not change from the perspective of position." Making perspectives explicit is the crucial step in performing DQ analysis to solve a comparative analysis problem. Once the notion of perspective is explicit, one can address questions like "Which perspective best suits a problem" and "What inferences are sound?" The answers are not as obvious as they might appear.

For example, consider the 'obvious' inference "Since it is going slower it will

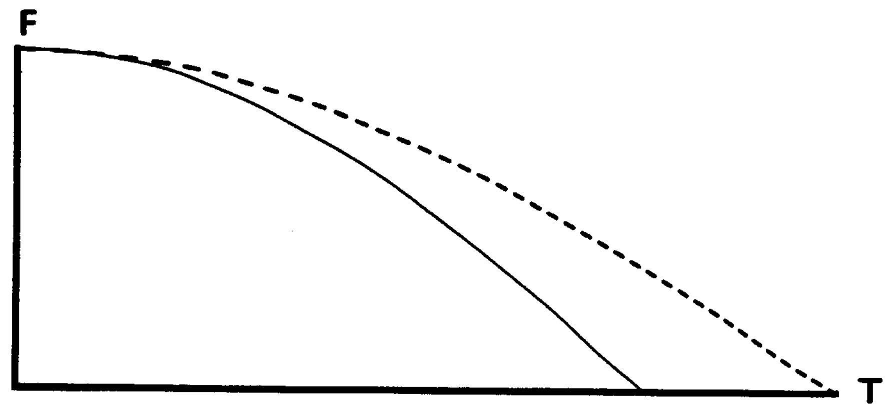

Figure 3: Actual Plot of Force versus Time

take longer to go the same distance." But what does it mean for the block to be going slower? From what perspective is velocity lower? If velocity were lower from the perspective of time, then the conclusion would indeed be obvious. But just as with the parameter force (figure 3), there are times when the perturbed velocity is not lower than it was in the original system. Once again, position is the correct perspective. In fact, as shown in section 3, the explanation is correct, but it would not necessarily be so if the perspective was some other parameter.

Reasoning about perspectives explicitly, and using sound rules of DQ analysis (section 3), the CA program has correctly generated the correct solution and an explanation like that shown above. Here is another example which it solves by using perspectives in a different way.

Example 2 *What happens to the maximum velocity if the initial displacement is increased?*

CA generates the justification which can be turned into English as follows:

> Since K and M haven't changed, the force on the block is the same for any position that the block used to pass through. So the acceleration is the same for any position. But since the initial displacement has been increased, the block will already be moving when it reaches the old initial position, where previously the block was stopped. Since the accelerations are the same from here on, and the block is already moving faster, it will keep on moving faster and will have a higher maximum velocity.

The rules which compose this reasoning are explained in section 3.

1.2 Changes in Behavioral Topology

The previous section showed how the explicit use of perspectives could determine the relative change of parameter values and time durations given an initial perturbation. However, sometimes the perturbation results in change of a more fundamental nature. Consider the heat exchanger shown in 4. Hot oil flows through the pipe losing heat to the cold water bath as it goes. Figure 5 shows a possible QSIM behavior that corresponds to the case when the hot oil reaches thermal equilibrium just as it exits from the pipe. (Remember that since this is a qualitative plot, the apparent slope does not imply that these functions are linear.) Let's pose a comparative analysis problem.

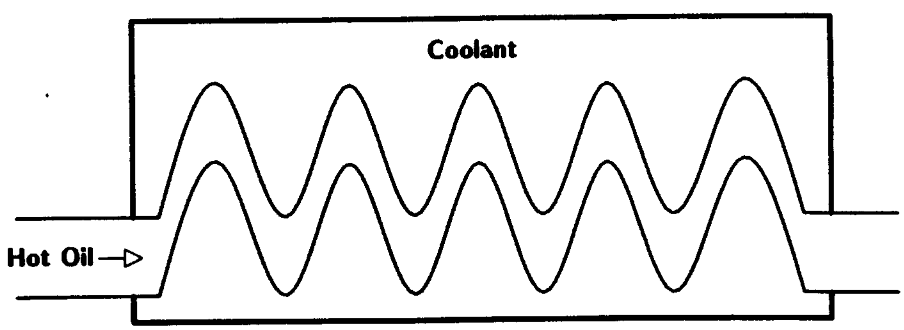

Figure 4: Hot Oil Flows Through Heat Exchanger

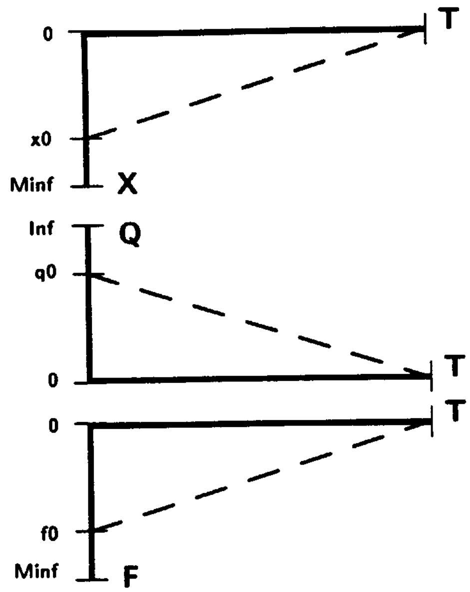

Figure 5: Initial Behavior of Heat Exchanger

Example 3 *What happens to the behavior of the heat exchanger if the thermal conductivity is increased?*

The answer is that the oil will more more quickly than before. And since the oil is flowing through the exchanger at the same rate, it must reach thermal equilibrium before leaving the pipe (figure 6). Thus, unlike the previous examples where the perturbation resulted only in continuous changes in various parameters, the perturbation of example 3 caused a discontinuous change: the previously cotemporaneous 'events' of thermal equilibrium and disgorgement from the pipe now happen at different times.

I call the switch from figure 5 to 6 a change in behavioral topology. Example 3 is a simple case of topological change: the initial behavior was inconsistent and a single new behavior was indicated. However, the situation isn't always so easy. Section 4 describes how perturbations can lead to multiple consistent behaviors and presents heuristics for determining the most likely resulting behavior.

1.3 Exaggeration

While most of this paper deals with the DQ solution technique to comparative analysis problems, it is worth noting that other qualitative techniques can solve similar problems. One such technique, called exaggeration [20], produces explanations that are completely different from those of the differential technique. Consider the question of example 1: "What happens to the period of oscillation

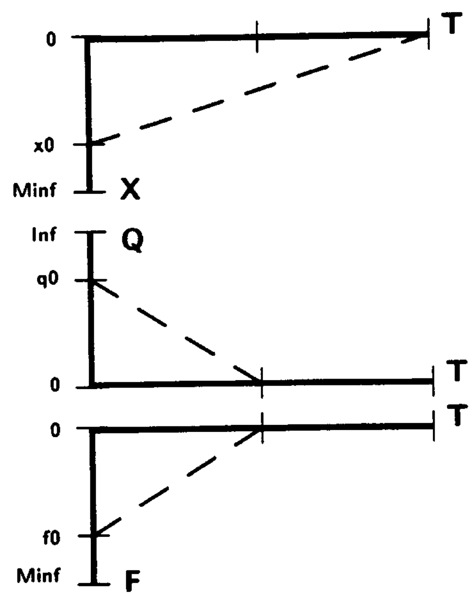

Figure 6: Perturbed Behavior of Heat Exchanger

if the mass of the block is increased?" Compare the exaggerated explanation with the one generated by DQ analysis.

> If the mass were infinite, then the block would hardly move at all. So the period would be infinite. Thus if the mass was increased a bit, the period would increase as well.

Exaggeration is a kind of asymptotic analysis—the perturbation is taken to the limit to make the effect more easily visible. Exaggeration is common in intuitive descriptions of physical behavior and appears quite powerful. As the example shows it often results in a concise explanation.

But exaggeration is subtle. It works only when the system responds monotonically to perturbations. Furthermore, it requires non-standard analysis to reason about infinity. It's quite easy to concoct a plausible exaggerated argument which is faulty, and a careful formalization of the technique is beyond the scope of this paper. See [20,22] for details.

1.4 Overview

The next section is foundational—it shows how perspectives are essential to a meaningful definition of relative change. Section 3 explains how the differential approach to comparative analysis can be implemented by a number of inference rules. The rules are proved sound, and their adequacy is discussed. Section

4 shows how to predict the effect of perturbations that change the behavioral topology of a system. In section 5 I discuss the relationship between comparative analysis and previous work in mathematics, engineering, and artificial intelligence. The paper concludes with a discussion of areas for future research and is followed by a brief appendix that contains details too technical for the main body of the text.

2 Preliminaries

As my formalism is based on that used by Kuipers for QSIM [15], I start out by summarizing his definitions.

Definition 1 *A* PARAMETER *is a reasonable function of time.*

See [15] for the actual definition of reasonable function; the intuition is that of continuity, continuous differentiability, and a finite number of critical points (places where its derivative is zero). Parameters are denoted by capital letters. Thus the velocity of a projectile might be described by the parameter, V, which is a function that maps time to velocity.

Definition 2 *Each parameter has an associated set of* LANDMARK VALUES *which is a subset of the range of the parameter. The landmark values always include (but aren't restricted to) zero, the values of the parameter at the beginning and ending times, and the values of the parameter at each of its critical points. A time, t, is a* DISTINGUISHED TIME POINT *of a parameter P if it is a boundary element of the set of times that $P(t) = p_i$ for some landmark value p_i.*

Landmark values are those values considered to be interesting to the human observer, and the times when these values are reached are of interest too. When a parameter becomes constant for an interval of time, then it will take on a landmark value for infinite number of time points. This is why the definition only considers the boundary times distinguished.

Definition 3 *A* SYSTEM *is a set of parameters that are related with a* STRUCTURAL DESCRIPTION *that consists of a finite set of qualitative differential equations defined using the following: time differentiation, addition, multiplication, and relation by monotonic functions.*

Kuipers' program, QSIM, takes a system and a set of initial values for each of the parameters and produces a set of possible behaviors for the system; the definitions below describe this behavioral output:

2.1 Qualitative Behavior

Definition 4 *Let $p_0 < \ldots < p_k$ be the landmark values of a parameter P. For any time t define the value of P at t as:*

$$QVAL(P,t) = \begin{cases} p_j & \textit{if } P(t) = \textit{landmark } p_j \\ (p_j, p_{j+1}) & \textit{if } P(t) \in (p_j, p_{j+1}) \end{cases}$$

Define the direction of P at t as:

$$QDIR(P,t) = \begin{cases} inc & if\ \frac{d}{dt}P(t) > 0 \\ std & if\ \frac{d}{dt}P(t) = 0 \\ dec & if\ \frac{d}{dt}P(t) < 0 \end{cases}$$

Define, $QS(P,t)$, the state of P at t, as the pair: $<QVAL(P,t), QDIR(P,t)>$

The qualitative state over the interval between two adjacent distinguished time points is defined similarly.

Definition 5 *For any parameter P, the* BEHAVIOR *of P is a sequence of states of P:*

$$\mathrm{QS}(P,t_0), QS(P,t_0,t_1), QS(P,t_1), \ldots, QS(P,t_{n-1},t_n), QS(P,t_n)$$

alternating between states at distinguished time-points, and states on intervals between distinguished time-points.

Recall that a system contains a set of parameters each with its own landmarks and distinguished time points.

Definition 6 *The* DISTINGUISHED TIME-POINTS *of a system are the union of the distinguished time-points of the parameters. Thus the state of a system changes whenever the state of any parameter changes. The* BEHAVIOR *of a system is thus a sequence of system-states alternating between distinguished time-points and intervals.*

To perform comparative analysis it is necessary to abstract away from specific times, since two different systems may have analogous behaviors, but change states at different times. This is where my formal treatment diverges from that of Kuipers.

Definition 7 *A parameter is said to reach a* TRANSITION *when its $QVAL$ changes to or from a landmark value. A system is said to reach a* TRANSITION *when any parameter transitions. Transitions only occur at distinguished time-points, and every distinguished time point marks a transition. It will prove useful to be able to refer to these transitions independent of the time at which they occur, thus the sequence of transitions for a behavior will be denoted by the set $\{\gamma_i\}$. Every behavior also has a* TIME FUNCTION, *$\mathcal{T}$, which takes transitions to the distinguished time-points when they occur.*

The intuition is that each γ marks an event which changes the state of the system. When comparing two behaviors, I match them up event by event and use the time functions to tell whether one system is changing faster or slower than the other.

2.2 Comparing Two Behaviors

To compare two behaviors, they must be distinguishable; I use the hat accent to denote the second behavior. Thus $\widehat{\mathcal{T}}$ denotes the time function of the second system, and $\widehat{F}(\widehat{\mathcal{T}}(\gamma_1))$ denotes the second system's value of F at the time of the first transition. To simplify the problem of of comparative analysis, I start by only comparing systems with identical structural descriptions whose behaviors are topologically equal, as defined below.

Definition 8 *The behaviors of two systems, S and $\widehat{S}$, are* TOPOLOGICALLY EQUAL *if they have the same sequence of transitions, $\gamma_0, \ldots, \gamma_k$, and forall i such that $0 \le i \le k$,*

$$\mathrm{QS}(S, \mathcal{T}(\gamma_i)) = \mathrm{QS}(\widehat{S}, \widehat{\mathcal{T}}(\gamma_i))$$

and forall i such that $0 \le i < k$,

$$\mathrm{QS}(S, \mathcal{T}(\gamma_i), \mathcal{T}(\gamma_{i+1})) = \mathrm{QS}(\widehat{S}, \widehat{\mathcal{T}}(\gamma_i), \widehat{\mathcal{T}}(\gamma_{i+1}))$$

The assumption of topological equality rules out possibilities like the block failing to make a complete oscillation if its mass was increased too much, but it does allow a certain pliability. If two behaviors are topologically equal, their respective sets of landmarks share the same ordinal relationships, but the underlying real values for the landmarks can be different.

Section 4 explains how this assumption can be relaxed, but even with it, the problem is nontrivial. Consider two oscillating spring-block systems. Even if the blocks have different mass and the spring constants differ, the two systems have topologically equal behavior. Yet the *relative* values of parameters such as period of oscillation may be different. These are the first changes that comparative analysis must determine.

Before I can explain the techniques for performing comparative analysis, I need to present a notation for describing the desired output. It's easy to compare the values of parameters at transition points:

Definition 9 *Given a parameter, F, and a transition γ_i, define the* RELATIVE CHANGE (RC) *of F at γ_i as follows:*

$$\begin{array}{ll} F{\Uparrow}_i & \text{if } |\widehat{F}(\widehat{\mathcal{T}}(\gamma_i))| > |F(\mathcal{T}(\gamma_i))| \\ F{\|}_i & \text{if } |\widehat{F}(\widehat{\mathcal{T}}(\gamma_i))| = |F(\mathcal{T}(\gamma_i))| \\ F{\Downarrow}_i & \text{if } |\widehat{F}(\widehat{\mathcal{T}}(\gamma_i))| < |F(\mathcal{T}(\gamma_i))| \end{array}$$

For example, if the two spring-block systems were both started with negative displacement and zero velocity (i.e., $X < 0$ and $V = 0$), their first transition would occur when X reached zero. This notation allows one to express that the second block is moving slower at the point of transition: $V{\Downarrow}_1$. It is important to distinguish the relative change notation from statements about values and derivatives. Even though $V \Downarrow_1$, QVAL($V, \mathcal{T}(\gamma_1)$) is positive, and QDIR($V, \mathcal{T}(\gamma_1)$) is *std*.

The curious reader may wonder at the use of absolute values in this definition. Relative change could also be defined by comparing signed values. I call the approach of definitions 9 and 11 MAGNITUDE SEMANTICS and the alternate approach SIGNED SEMANTICS. The two approaches are theoretically equivalent. However, since magnitude semantics appears somewhat more natural and simplifies various proofs, it is the default for the rest of the paper. In the places where signed semantics proves advantageous, it will be mentioned explicitly.

2.3 Comparing Two Behaviors over Intervals

It turns out to be somewhat more complicated to compare two behaviors over the intervals between transitions. What does it mean to says that one curve is lower than another over an interval? To do pointwise comparison, some notion of corresponding points is necessary.

The intuition for the requisite comparison is displayed in the explanation of spring behavior that was presented in section 1.1.

> If the mass of the block increases, the force on the block is the same....

Yet this doesn't mean force is invariant as a function of time—that isn't true. Consider the time when the small block is at its rest position; the spring applies no force. But since the large block is moving more slowly, it won't have reached the rest position and so there will be a force applied.

What the statement means is that force is invariant as a function of position. For every position that the block occupies, force is equal in the two systems, even though the two blocks occupy the positions at different times. Although parameters are defined as functions of time, they often need to be compared from the perspective of other parameters. Here it proved advantageous to consider force as a function of position. Although people understand arguments that leave these changes of variable implicit, the notion must be made precise and explicit if computers are to perform comparative analysis. The notion of perspective is foundational.

Definition 10 *A parameter, X, is called a* COVERING PERSPECTIVE *over a transition interval (γ_i, γ_{i+1}) when the following three conditions hold:*

1. *QDIR*$(X, \mathcal{T}(\gamma_i), \mathcal{T}(\gamma_{i+1})) \neq$ *std*
2. $X\|_i$
3. $X\|_{i+1}$

When just the first condition holds, X is called a PARTIAL PERSPECTIVE.

When a parameter, X, is a partial perspective, it is strictly monotonic so its inverse X^{-1} exists. This means that it is possible to reparameterize any other parameter, F, by composing it with the inverse:

$$F_X(x) = F(X^{-1}(x))$$

When X is a covering perspective, then F_X and $\widehat{F_X}$ have the same domain. Covering perspectives will prove especially important in the inference rules of section 3.

Definition 11 *Given a parameter F, a partial perspective X, and a transition interval (γ_i, γ_{i+1}), let F_X denote F as a function of X. Let U be the intersection of the domains of F_X and $\widehat{F_X}$*

$$U = (X(\mathcal{T}(\gamma_i)), X(\mathcal{T}(\gamma_{i+1}))) \cap (\widehat{X}(\widehat{\mathcal{T}}(\gamma_i)), \widehat{X}(\widehat{\mathcal{T}}(\gamma_{i+1})))$$

Define the RELATIVE CHANGE (RC) *of F over (γ_i, γ_{i+1}) from the* PERSPECTIVE *of X as follows:*

$$\begin{array}{ll} F{\Uparrow}^{X}_{(i,i+1)} & \textit{if } \forall x \in U \ |\widehat{F_X}(x)| > |F_X(x)| \\ F{\|}^{X}_{(i,i+1)} & \textit{if } \forall x \in U \ |\widehat{F_X}(x)| = |F_X(x)| \\ F{\Downarrow}^{X}_{(i,i+1)} & \textit{if } \forall x \in U \ |\widehat{F_X}(x)| < |F_X(x)| \end{array}$$

In other words, force is $\|$ from the perspective of position, if for all positions that are assumed in both simulations ($\forall x \in U$) the corresponding forces are equal. The definition of partial perspective says when is it possible to use a parameter as a perspective; section 3 addresses the question when is it useful to do so.

2.4 Time as a Perspective

Although comparisons of parameters that have been reparameterized by perspectives are more common, sometimes is is useful to compare via corresponding times. To keep notation consistent, I will call this 'using time as a perspective.' The goal is to come up with a meaningful definition for $P{\Uparrow}^{\mathcal{T}}_{(i,i+1)}$ and the other RC values.

One problem is that the duration of the two time intervals might be different. If so time acts as a partial perspective—one quantifies only over time in the shortest interval. Another problem is that the two transition intervals might start at different times; in fact one interval might end before the other starts, e.g., $\widehat{\mathcal{T}}(\gamma_i) > \mathcal{T}(\gamma_{i+1})$. The solution is to align the intervals before quantifying.

Definition 12 *Given a parpameter P and an interval (γ_i, γ_{i+1}). Let $U = (0, d)$ where $d = MIN(T(\gamma_{i+1}) - T(\gamma_i), \hat{T}(\gamma_{i+1}) - \hat{T}(\gamma_i))$. Define the* RELATIVE CHANGE (RC) *of P over (γ_i, γ_{i+1}) from the* PERSPECTIVE OF TIME *as follows:*

$$
\begin{array}{ll}
P\uparrow^{T}_{(i,i+1)} & \text{if } \forall t \in U \ |\hat{P}(\hat{T}(\gamma_i) + t)| > |P(T(\gamma_i) + t)| \\
P\|^{T}_{(i,i+1)} & \text{if } \forall t \in U \ |\hat{P}(\hat{T}(\gamma_i) + t)| = |P(T(\gamma_i) + t)| \\
P\downarrow^{T}_{(i,i+1)} & \text{if } \forall t \in U \ |\hat{P}(\hat{T}(\gamma_i) + t)| < |P(T(\gamma_i) + t)|
\end{array}
$$

3 Differential Qualitative Analysis

This section presents a number of rules for computing and manipulating RC values, describes how the rules were incorporated into a computer program, and evaluates the program's performance.

- The duration rule formalizes "distance equals rate times time."
- The interval derivative rule expresses the relationship between one derivative and another, e.g., "more acceleration leads to higher velocity."
- The transition derivative rule predicts the final value of a derivative like velocity.
- The self reference rule says that every parameter appears unchanged from its own perspective.
- The perspective flipping rule allows a reasoner to change perspectives.
- The transition and interval constant rules show the relationship between constants and RC values.
- The end of time rule says that other things being equal a parameter changes more, the longer it is changing.
- The one's own derivative rule predicts what happens when a parameter is defined in terms of itself.
- The multiplication rule demonstrates that the familiar rules of qualitative arithmetic apply to RC values as well as derivatives.

Each of the rules are presented as theorems since they are proven sound. For simplicity, however, only the interesting and difficult proofs have been included in this paper. The rules have been implemented as part of CA, a ZETALISP program which solves comparative analysis problems using DQ analysis. CA uses a constraint propagator to derive implications of these rules. The resulting dependency structure can be translated into an English explanation or used by an explanation based generalizer. Although CA is incomplete (there are some problems it for which it terminates without solving), it does answer and intuitively explain a large class of problems. Because the rules have been proven correct, CA is guaranteed to reach only sound conclusions.

3.1 Duration Rule

This rule is the basis for the very powerful inference: distance equals rate times duration. If the rate is slower in the second simulation, then it will take longer to go the same distance. Although this may seem obvious, perspectives are required to make precise the notion of 'rate is slower'; this makes it subtle. Before I can state the theorem, the notion of distance must be made clear.

Definition 13 *Let X be a parameter which is increasing and positive (or decreasing and negative)*[2] *over the transition interval (γ_i, γ_{i+1}). Define* DISTANCE-BY *X over (γ_i, γ_{i+1}) as the relative change of the distance traveled over X over the interval as shown in the following table:*

		Starting RC Value	
	$\Uparrow$	$\|$	$\Downarrow$
Ending $\Uparrow$	*?*	$\Uparrow$	$\Uparrow$
RC $\|$	$\Downarrow$	$\|$	$\Uparrow$
Value $\Downarrow$	$\Downarrow$	$\Downarrow$	*?*

Note that the parameter X has a double purpose in this theorem: it has V as its time derivative, and it is also the perspective from which V is seen to $\Downarrow$. In the following, it may be helpful to think of V as velocity, and X as position.

Proposition 1 Duration Rule
*Let V and X be parameters such that X is a partial perspective over (γ_i, γ_{i+1}). Given $V = \frac{d}{dt}X$, $V\Downarrow^X_{(i,i+1)}$, and $\neg$*DISTANCE-BY*$X\Downarrow_{(i,i+1)}$ then $\widehat{T}(\gamma_{i+1}) - \widehat{T}(\gamma_i) > T(\gamma_{i+1}) - T(\gamma_i)$, i.e. the duration of (γ_i, γ_{i+1}) will increase.*

Proof: Note that the proof is not obvious: $V_X \neq \frac{dx}{dt}$. I prove the case in which DISTANCE-BY$X||_{(i,i+1)}$. This is equivalent to requiring X to be a covering perspective. Let $a = X(T(\gamma_i))$ and $b = X(T(\gamma_{i+1}))$. Since X is a covering perspective, X has an inverse function taking position to time:

$$X^{-1} : (a,b) \rightarrow (T(\gamma_i), T(\gamma_{i+1}))$$

The function $\widehat{X}^{-1}$ also exists, has the same domain, and a possibly different range: $(\widehat{T}(\gamma_i), \widehat{T}(\gamma_{i+1}))$. By definition $V\Downarrow^X_{(i,i+1)}$ means:

$$|\widehat{V}(\widehat{X}^{-1}(x))| < |V(X^{-1}(x))| \quad \forall x \in (a,b)$$

Consider the case[3] where $V > 0$; this implies that all values of $\widehat{V}$ are greater than zero because otherwise the two systems would have different transitions, violating the topological equality assumption. This means that:

$$0 < \widehat{V}(\widehat{X}^{-1}(x)) < V(X^{-1}(x)) \quad \forall x \in (a,b)$$

So:

$$\frac{1}{\widehat{V}(\widehat{X}^{-1}(x))} > \frac{1}{V(X^{-1}(x))} > 0 \quad \forall x \in (a,b)$$

[2] A similar definition is made for the cases of increasing/negative and decreasing/positive. This definition would be simpler to express in signed semantics.

[3] The case where $V < 0$ is similar; there is no case where $V = 0$ because then X would not reach a transition.

So:

$$\int_a^b \frac{1}{\widehat{V}(\widehat{X}^{-1}(x))}\,dx > \int_a^b \frac{1}{V(X^{-1}(x))}\,dx > 0$$

But by the chain rule, the time derivative of X^{-1} at x is $\frac{1}{V(X^{-1}(x))}$. So:

$$\widehat{X}^{-1}(b) - \widehat{X}^{-1}(a) > X^{-1}(b) - X^{-1}(a) > 0$$

Thus: $\widehat{\mathcal{T}}(\gamma_{i+1}) - \widehat{\mathcal{T}}(\gamma_i) > \mathcal{T}(\gamma_{i+1}) - \mathcal{T}(\gamma_i)$. In other words, the duration of the interval increases. □

It would be nice if one could show that the duration rule was sound if the premise was weakened to have $V\Downarrow^P_{(i,i+1)}$ for some arbitrary covering perspective P. However, the following proposition shows that this is false; just because $P||^X_{(i,i+1)}$ for a perspective X doesn't mean that there doesn't exist some other perspective Z such that $P\Uparrow^Z_{(i,i+1)}$.

Proposition 2 Non–Uniqueness
Given a system with parameters P, X, Y, and Z such that X, Y and Z are covering perspectives over (γ_i, γ_{i+1}), then it is possible that $P\Uparrow^X_{(i,i+1)}$ and $P||^Y_{(i,i+1)}$ and $P\Downarrow^Z_{(i,i+1)}$.

The example shown in figure 7 illustrates the proof by construction. The thin lines indicate the values of the first system while the dotted lines indicate the value of the second system. The first row shows that from the time perspective the behavior of P doesn't change. The second row shows the relative change of the perspectives. The third row depicts P_X, P_Y and P_Z.

Although this aspect of RC values may seem strange, it is actually inevitable. After all, everything is relative to one's perspective. Imagine a machine which hourly logs the linearly increasing concentration of alcohol in a fermentation tank. It produces the following sequence of measurements: 0.02, 0.04, 0.06, 0.08, etc. But in the identical tank nearby, the logging machine has a defective motor which runs too slowly and delays the measurements. Although the fermentation is proceeding at the same pace in both tanks, the second log will read: 0.03, 0.06, 0.09, 0.12, etc. Thus the plant inspector, who only sees the alcohol-time curve from the perspective of the logging device, might think that second tank was fermenting more quickly even though the only real change was a slowdown in the speed of the timing motor.

3.2 Derivative Rules

These rules connect parameters that are time derivatives. The first works over intervals and the second predicts RC values at interval endpoints. The intuition behind the first is: if a parameter is $||$ at the start of an interval, but its derivative is $\Downarrow$ over the interval, then the parameter must be $\Downarrow$ over the interval. As always,

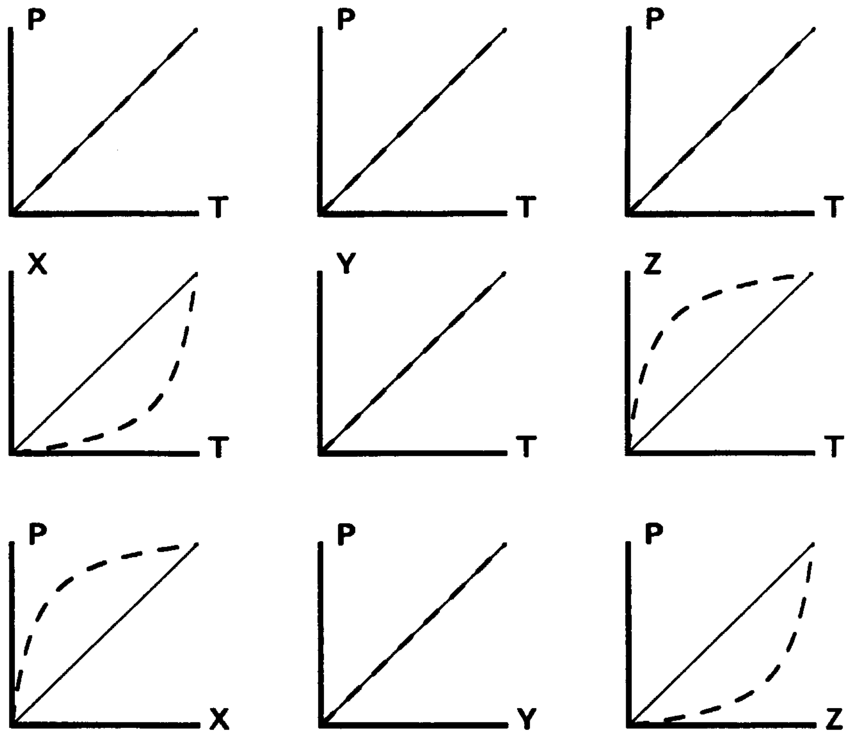

Figure 7: $P\Uparrow^{X}_{(0,1)}$ $yet P\|^{Y}_{(0,1)}$ $and P\Downarrow^{Z}_{(0,1)}$

the ubiquity of perspectives complicates the matter. Note the special role of X both as perspective and second integrand of A.

Proposition 3 Interval Derivative Rule
Let A, V, and X be parameters such that $A = \frac{d}{dt}V$, $V = \frac{d}{dt}X$, and X is a covering perspective over (γ_i, γ_{i+1}). Furthermore let A and V be positive over the interval (γ_i, γ_{i+1}). If $\neg V\Uparrow_i$, $A\Downarrow_i$, and $A\Downarrow^{X}_{(i,i+1)}$. Then $V\Downarrow^{X}_{(i,i+1)}$.

Proof: The chain rule makes this rule considerably harder to prove than the duration rule. It suffices to show that there exists some position such that $|\widehat{V}| < V$ for all positions up to and including this position. Once it is known that V goes down, the same argument can be used to show that it continues to go down. Thus it will stay down until γ_{i+1} is reached.
Let

$$\dot{\tau}(x) = \frac{dt}{dx} = \frac{1}{V(X^{-1}(x))}$$

Let

$$\ddot{\tau}(x) = \frac{d^2t}{dx^2}$$

A can be expressed as a function of X

$$A(X^{-1}(x)) = \frac{-\ddot{r}(x)}{(\dot{r}(x))^3}$$

Since $A\Downarrow_i$ and $A\Downarrow^X_{(i,i+1)}$, it is the case that for all x in the half open interval $[a,b)$

$$\frac{\widehat{\ddot{r}}(x)}{(\widehat{\dot{r}}(x))^3} > \frac{\ddot{r}(x)}{(\dot{r}(x))^3} \tag{1}$$

Because $\neg V\Uparrow_i$ and since V is positive,

$$\widehat{\dot{r}}(a) \geq \dot{r}(a) \geq 0 \tag{2}$$

Substituting (2) in the denominator of (1) gives

$$\frac{\widehat{\ddot{r}}(a)}{(\widehat{\dot{r}}(a))^3} \geq \frac{\ddot{r}(a)}{(\dot{r}(a))^3} \geq \frac{\ddot{r}(a)}{(\widehat{\dot{r}}(a))^3}$$

So

$$\widehat{\ddot{r}}(a) > \ddot{r}(a) \tag{3}$$

And by continuity, equation (3) holds over a half open interval which may be written as $[a,c)$ for some c. This implies that the equation holds over the closed interval $[a,d]$ where $d = a + \frac{c-a}{2}$. But by the definition of $\ddot{r}$, for any $x_0 \in [a,d]$

$$\dot{r}(x_0) = \dot{r}(a) + \int_a^{x_0} \ddot{r}(x)\,dx$$

So for all $x \in (a,d]$

$$\widehat{\dot{r}}(x) > \dot{r}(x)$$

So for all $x \in (a,d]$

$$\frac{1}{\widehat{\dot{r}}(x)} < \frac{1}{\dot{r}(x)}$$

Thus by the definition of $\dot{r}$, for all $x \in (a,d]$

$$\widehat{V}(\widehat{X}^{-1}(x)) < V(X^{-1}(x))$$

So $V\Downarrow^X_{(i,i+1)}$ $\square$

Above I pointed out the special role of X both as perspective and second integrand of A. It is natural to ask if the interval derivative rule is true for arbitrary perspectives. Unfortunately, it is not. Appendix A provides a counter-example which makes this point.

The interval derivative rule has an important corollary which predicts the value of the middle derivative, V, at the transition ending the interval. The intuition is threefold:

- If the object is accelerating slower, then its terminal velocity will be smaller.
- If the object is accelerating at the same rate but starts with a slower initial velocity, its terminal velocity will be smaller.
- If the object accelerates for a shorter distance, then it will finish going slower.

Proposition 4 Transition Derivative Rules
Let A, V, and X be parameters such that $A = \frac{d}{dt}V$, $V = \frac{d}{dt}X$, X is a partial perspective over (γ_i, γ_{i+1}), and both A and V are positive over the interval. If one of the following conditions is true,

- $V\Downarrow_i$ *and* $A\|^X_{(i,i+1)}$ *and* DISTANCE-BY$X\|_{(i,i+1)}$
- $(A\Downarrow_i \wedge A\Downarrow^X_{(i,i+1)})$ *and* $V\Downarrow_i$ *and* DISTANCE-BY$X\|_{(i,i+1)}$
- DISTANCE-BY$X\Downarrow_{(i,i+1)}$ *and* $V\Downarrow_i$ *and* $A\|^X_{(i,i+1)}$

then $V\Downarrow_{i+1}$.

The rule is quite a mouthful, but that is simply because it is very general.

3.3 Perspective Rules

These rules deal with establishing RC values for perspectives and switching between them. The first is very simple, but turns out to be quite important. The intuition is that if the plant manager was foolish enough to try and use the logging devices to log their own speed, he wouldn't get a useful result. Both the normal and slow machines would record that they turned one full revolution during each revolution of the timing motor.

Proposition 5 Self Reference Rule
For any parameter P, if P is a covering perspective over (γ_i, γ_{i+1}) then $P\|^P_{(i,i+1)}$.

The perspective flipping rules switches between perspectives. The intuition is that flipping perspectives (i.e., X^P to P^X) flips $\Uparrow$ to $\Downarrow$ if both parameters are positive and increasing over the interval.

Proposition 6 Perspective Flipping Rule
If the parameters X and P are valid perspectives over (γ_i, γ_{i+1}), the sign of X equals the sign of P over the interval, and $X\Uparrow^P_{(i,i+1)}$, then:

$$\begin{array}{ll} P\Uparrow^X_{(i,i+1)} & \text{if } QDIR(X, \mathcal{T}(\gamma_i), \mathcal{T}(\gamma_{i+1})) \neq QDIR(P, \mathcal{T}(\gamma_i), \mathcal{T}(\gamma_{i+1})) \\ P\Downarrow^X_{(i,i+1)} & \text{if } QDIR(X, \mathcal{T}(\gamma_i), \mathcal{T}(\gamma_{i+1})) = QDIR(P, \mathcal{T}(\gamma_i), \mathcal{T}(\gamma_{i+1})) \end{array}$$

If the sign of X is the opposite of the sign of P then the RC values are reversed.

Proof: I will prove the case where both X and P are increasing; the other cases are almost identical. Let $a = X(\mathcal{T}(\gamma_i))$, and $b = X(\mathcal{T}(\gamma_{i+1}))$. For an arbitrary $x \in (a, b)$ $\exists p$ such that $X(P^{-1}(p)) = x$ because P is a covering perspective, and thus onto. Let $t_1 = \widehat{P}^{-1}(p)$, and let

$$\widehat{x} = \widehat{X}(t_1) = \widehat{X}(\widehat{P}^{-1}(p)$$

By the definition of $X{\Uparrow}^{P}_{(i,i+1)}$ it follows that $\widehat{x} > x$. Let $t_0 = \widehat{X}^{-1}(x)$. Since X is increasing $t_0 < t_1$. Again because P is onto, $\exists \widehat{p}$ such that $\widehat{P}^{-1}(\widehat{p}) = t_0$ so $\widehat{X}(\widehat{P}^{-1}(\widehat{p})) = x$. Now, $\widehat{p} < p$ because

$$\widehat{P}^{-1}(\widehat{p}) = t_0 < t_1 = \widehat{P}^{-1}(p)$$

and $\widehat{P}$ is increasing. But this means that

$$\widehat{P}(\widehat{X}^{-1}(x)) < P(X^{-1}(x))$$

and since x was arbitrary, it follows that $P{\Downarrow}^{X}_{(i,i+1)}$ □

3.4 Constants

Frequently a system will contain a few constant parameters whose values never change. The following rules are a simple way to express relationships between constants in the notation of comparative analysis. The intuition is that since perspectives just scale time, and constants don't change over time, all perspectives agree on the behavior of constants. If there was no fermentation happening in either vat (i.e. the alcohol concentration was constant in both vats), and the concentration of alcohol was higher in vat two, then both logging devices would agree on this even though their timing motors differed.

Proposition 7 Transition Constant Rule
If a parameter K is a constant over (γ_i, γ_{i+1}), and $K{\Uparrow}i$ then $K{\Uparrow}_{i+1}$.

Proposition 8 Interval Constant Rule
If a parameter K is a constant over (γ_i, γ_{i+1}), and $K{\Uparrow}_i$ then for all parameters P, if P is a covering perspective over the interval (γ_i, γ_{i+1}), then $K{\Uparrow}^{P}_{(i,i+1)}$.

3.5 Rules with Time as a Perspective

It is very common for one parameter to be the derivative of another with respect to time. When it is possible to reason about these relations from the perspective of time, greater power is achieved because the chain rule doesn't interfere as it does in the derivative rule. The only drawback is the fact that these rules are less frequently applicable.

The first rule says that if the a parameter is $||$from the perspective of time, and the duration of the interval is increasing, then the parameter will have changed more by the end of the interval.

Proposition 9 The End of Time Rule
Let X be a parameter such that $X\|_i$ and $X\|^{\mathcal{T}}_{(i,i+1)}$. Let s be the sign of X over the transition interval (γ_i, γ_{i+1}) and d be the sign of X's derivative. If the duration of (γ_i, γ_{i+1}) is $\Uparrow$, then

$$\begin{array}{ll} X\|_i & \textit{if } d = 0, \textit{ otherwise} \\ X\Uparrow_i & \textit{if } s = d \\ X\Downarrow_i & \textit{if } s \neq d \end{array}$$

The proof of this lemma is trivial and thus omitted, but it should be noted that it is easier to express using signed semantics. The second rule is used for determining a parameter RC value from the perspective of time. It applies whenever the time derivative of a parameter is a linear function of the parameter.

Proposition 10 One's Own Derivative Rule
Let X, V, and K be parameters such that $V = \frac{d}{dt}X$, $V = \text{MULT}(X, K)$, and K is a negative constant. If $V(\mathcal{T}(\gamma_i)) \neq 0$ and $\|_i$ and $K\Uparrow^{\mathcal{T}}_{(i,i+1)}$ then $X\Downarrow^{\mathcal{T}}_{(i,i+1)}$.

3.6 Rules from Qualitative Arithmetic

Research in qualitative simulation [2,7,26,15] has developed constraints on derivative values for parameters in ADD, MULT, and monotonic function constraints. For example, if $X \times Y = Z$ and the derivatives of X and Y are positive, then Z must have positive derivative as well. These rules can be generalized to include RC values at transition points and over intervals. Here, I present just the rule for a MULT constraint at a transition point.

Proposition 11 Multiplication Rule
If X, Y, and Z are parameters which are related by the constraint, $Z = \text{MULT}(X, Y)$, then the following table displays the possible RC values for Z at a transition point:

		Y		
		$\Uparrow_i$	$\|_i$	$\Downarrow_i$
	$\Uparrow_i$	$\Uparrow_i$	$\Uparrow_i$	?
X	$\|_i$	$\Uparrow_i$	$\|_i$	$\Downarrow_i$
	$\Downarrow_i$	?	$\Downarrow_i$	$\Downarrow_i$

The rule for the ADD constraint is similar, but complex to write using magnitude semantics.

3.7 Implementation

To test the theory of DQ analysis, a program called CA has been written on a Symbolics lisp machine. When a user selects an example, CA runs QSIM [15] on the example to produce a set of qualitative behaviors for the example. The user selects a behavior and also a set of initial RC perturbations. CA translates the QSIM behavior and perturbations into ARK[4] assertions. At this point ARK forward chains using the propositions described earlier in this section.

Each of these propositions is implemented as an ARK rule or more than one if the proposition used disjunction or negation. For example, the duration rule (proposition 1) is encoded as the three ARK rules of figure 8. The various definitions and propositions require about sixty ARK rules.

```
(⇒ (AND (D/DT ?x ?v)                        ; ?v is the derivative of ?x
        (DISTANCE-BY ?x (?start ?end) deq)  ; ?x travels the same distance
        (RC ?v (?start ?end) ?c (P- ?x))    ; the RC of ?v is ?c from
        (OPPOSITE-RC ?c ?oc))               ;     the partial persp. of ?x
   (DURATION (?start ?end) ?oc)             ; if ?c is ⇑, ?oc is ⇓
   duration-rule1)

(⇒ (AND (D/DT ?x ?v)
        (DISTANCE-BY ?x (?start ?end) ?oc)  ; if ?x travels ?oc distance
        (RC ?v (?start ?end) ?c (P- ?x))    ; and V's RC agrees
        (OPPOSITE-RC ?c ?oc))
   (DURATION (?start ?end) ?oc)             ; then the duration is ?oc
   duration-rule2)

(⇒ (AND (D/DT ?x ?v)
        (DISTANCE-BY ?x (?start ?end) ?oc)  ; if ?x travels 'less' distance
        (RC ?v (?start ?end) deq (P- ?x)))  ; and V doesn't change
   (DURATION (?start ?end) ?oc)             ; then the duration is 'less'
   duration-rule3)
```

Figure 8: Propositions Are Encoded Directly Into ARK Rules

The simplicity of the transformation from proposition to ARK code provides confidence in the soundness of the implementation. And the fact that most rules get used in each explanation, establishes their utility.

Since it is an initial prototype, CA makes no use of control rules. All possible forward chaining inferences are made using every possible perspective. Despite this, computation rarely exceeds a minute on any of the problems tested. If

[4] ARK is a descendant of AMORD [3] implemented by Howie Shrobe and others.

larger problems were to be attempted, some form of control would be desirable. Backward chaining from a goal pattern might increase efficiency. There appears to be no reason why the schemes of [4,24] could not be applied. Possible heuristics include preferential investigation of certain perspectives and avoidance of certain computationally explosive rules like the perspective-flip rule.

Another technique to speed up reasoning is explanation based generalization [16,5]. Following the approach of [13], I implemented a postprocessing learning routine that takes CA explanations and produces new ARK rules which may be added to the ones presented above. While these new rules are independent of any particular domain (i.e., springs), they are optimized to solve a specific class of comparative analysis problems. Less general than the rules presented above, the new rules are considerably more general than the specific explanation from which they are derived. Although I have completed the EBG implementation, the empirical evaluation of EBG's ability to increase DQ processing efficiency remains as an area for future research.

3.8 Differential Analysis Suffices for Most Examples

Since ARK maintains justifications for all its assertions, it is possible to generate explanations for CA's conclusions. Consider the spring/block example. The question here is: "What happens to the period of spring oscillation if the mass of the block is increased?" The system is defined in terms of six parameters: spring constant $-K$, mass M, position X, velocity V, acceleration A, and force F obeying the following equations:

$$\begin{aligned} A &= \tfrac{d}{dt} V \\ V &= \tfrac{d}{dt} X \\ F &= \text{MULT}(M, A) \\ F &= \text{MULT}(-K, X) \\ \tfrac{d}{dt} M &= std \\ \tfrac{d}{dt} - K &= std \end{aligned}$$

The initial conditions are specified as follows: $M(0) > 0,\ -K(0) < 0,\ V(0) = 0,$ and $X(0) = x_0 < 0$. Since energy conservation is not made explicit in the equations, QSIM produces several possible behaviors for this system. Although comparative analysis could be done on any of the behaviors, I assume in this example that the user selects the interpretation corresponding to stable oscillation.

Now the user selects the perturbation. Because some parameters depend on one another, not all parameters may be perturbed. The situation is analogous to the problem of specifying a unique solution to a differential equation where values must be given for the independent parameters and a set of boundary conditions provided. In this example, M and $-K$ are independent, while values for X and V are needed as boundary conditions. Thus to specify a comparartive analysis problem, these four parameters need to be given initial RC values.[5] For this example, the perturbation consists of the following initial RC values: $M\Uparrow_0$, $-K\|_0$, $V\|_0$, and $X\|_0$.

Given this input, CA correctly deduces that the block will take longer to reach the rest position ($X = 0$) from its original negative stretched position. Figure 9 shows the explanation that CA generates; this is created by throwing away all perspective information once computation is finished. I have annotated the explanation with the names of rules used in each step.

Assuming M is increased:	
X doesn't change and	*(self-reference rule)*
K doesn't change and	*(interval constant rule)*
F equals -K times X	
So F doesn't change.	*(multiplication rule)*
and	
M increases and	*(interval constant rule)*
F equals M times A	
So A decreases.	*(multiplication rule)*
So V decreases.	*(derivative rule)*
So the time duration increases.	*(duration rule)*

Figure 9: CA Generated Explanation for Spring with Heavier Block

At present CA has been tested for multiple perturbations on over a dozen examples including the RC circuit shown in figure 10. While it always terminates and never produces an incorrect answer, CA doesn't necessarily deduce RC values for every parameter.

3.9 Differential Analysis is Incomplete

As is explained in the sections below, different types of ambiguity are the cause for the incompleteness of DQ analysis. Yet DQ analysis handles ambiguity differently from other forms of qualitative reasoning. For example, when QSIM is faced with ambiguity about a parameter's value, it branches, spawning perhaps three new behaviors: one with the parameter equal to a landmark value, one

[5]The choice of these four parameters is somewhat arbitrary. Mathematically, it would be equally reasonable to choose A instead of X, but this does not make physical sense; it seems intuitively impossible to directly affect acceleration. Since there is no way to deduce this from the differential equation model, it is esential for the person who constructs the model to annotate the structural description with the list of 'causally primitive' parameters — in this case, the four listed above.

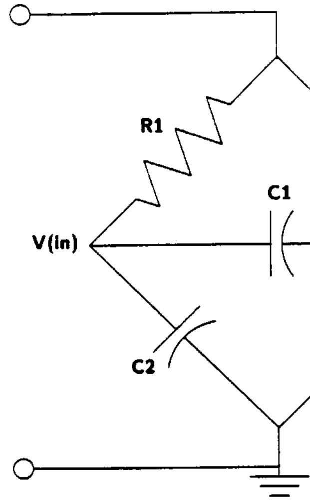

Figure 10: An RC Circuit Based on the Wheatstone Bridge

greater and one less. QSIM can do this because the nature of inequality guarantees that either $A < B$, $A = B$, or $A > B$. While this is true for RC values at transition points, it is not true for RC values over intervals.

Proposition 12 Non–Exhaustivity
Given two parameters, V and P, such that P is a covering perspective over an interval (γ_i, γ_{i+1}), it is not necessarily the case that one of $V\Uparrow^P_{(i,i+1)}$, $V\|^P_{(i,i+1)}$, or $V\Downarrow^P_{(i,i+1)}$ holds.

Proof: Appendix A provides an example which proves this statement □

Thus unlike qualitative simulators, DQ analysis can not branch when faced with uncertainty, it simply acts mute. The following sections explain the three factors that can cause DQ analysis to fail to predict all of the relative changes in a perturbed system: ambiguous questions, ambiguity resulting from the qualitative arithmetic, and the lack of a useful perspective.

3.9.1 Ambiguous Questions

Some questions simply don't contain enough information. For example: "What would happen to the period of oscillation, if the mass of the block was heavier and the spring was more stiff?" There is no answer to this question because it is inherently ambiguous. The increased mass tends to increase the period, but the increased spring constant tends to decrease it. Thus the duration might increase, decrease or remain unchanged.

3.9.2 Ambiguity Introduced by Qualitative Arithmetic

Since DQ analysis uses the same qualitative arithmetic utilized by other forms of qualitative reasoning, it should not be surprising that ambiguity causes a problem here as well. The problem is rooted in the fact that qualitative values (of which RC values are an instance) do not form a group over addition [18]. As a result, unique inverses do not always exists and it is frequently impossible to determine the qualitative value of a parameter.

For example, consider the spring/block system of the last section. DQ analysis correctly predicts that the block will take longer to reach the first transition, the block's rest position. But the period of oscillation requires four transitions: starting from a negative initial position, X moves to zero, then to a positive maximum, then to zero, and finally to its original position. Because of ambiguity in the extreme positions of X, DQ analysis can make no prediction about duration of these last three transition intervals. Why is this? Because of the qualitative arithmetic, it is impossible to show that $X||_2$, i.e., that X sweeps out the same distance when the mass is increased. Because of this, X is not known to be a covering perspective so the derivative and duration theorems can not be used. Thus there is no way to determine the RC value for the whole period.

This problem is directly analogous to QSIM's prediction of spurious behaviors [15]. Given a Hooke's law description of the spring/block, QSIM produces many possible behaviors in addition to the correct description of stable oscillation. Furthermore, the DQ problem can be alleviated in the same way that Kuipers caused QSIM to disregard behaviors other than stable oscillation—by augmenting the structural description with equations describing conservation of energy. Now CA can deduce that since potential energy is equal to force times distance, increasing the block's mass leaves total energy unchanged. This allows it to recognize X as a covering perspective and deduce that the duration increases for each of the period's four transition intervals.

3.9.3 No Useful Perspective

Other questions are even more difficult to answer: "What would happen to the period of oscillation if the initial displacement is increased?" Since people have trouble with this question, it should not be surprising that DQ analysis cannot answer the question either. In fact, the answer is "period does not change", but the only way to show this is to solve the differential equation for an equation for period and notice that it is independent of amplitude. The difficulty is rooted in the fact that ***no useful perspective exists*** to provide a handle on the problem. There is no system parameter P such that $V||^P_{(0,1)}$. Clearly X won't work as a perspective, since it doesn't sweep out the same range in the two cases. In fact, it is easy to prove that no artificial perspective could satisfy the equation.

Proposition 13 *Given the definition of V as specified above for the spring/block example with $X{\Uparrow}_0$, let $t_1 = \mathcal{T}(\gamma_1)$ and $\widehat{t_1} = \widehat{\mathcal{T}}(\gamma_1)$. There are no continuous, real valued, functions $P, \widehat{P}$ such that*

$$\begin{aligned} P(0) = \widehat{P}(0) = p_0 \ \wedge \\ P(t_1) = \widehat{P}(\widehat{t_1}) = p_1 \ \wedge \\ \widehat{V}(\widehat{P}^{-1}(p)) = V(P_{-1}(p)) \ \forall p \in (p_0, p_1) \end{aligned}$$

Proof: Since $X{\Uparrow}_0$, initial potential energy is higher in the perturbed system, so kinetic energy is greater at γ_1. This means that $V{\Uparrow}_1$, i.e.,

$$\widehat{V}(\widehat{t_1}) = \widehat{v_1} > v_1 = V(t_1)$$

Because $\widehat{V}$ and V are continuous

$$\lim_{t \to \widehat{t_1}} \widehat{V}(t) = \widehat{v_1}$$

and

$$\lim_{t \to t_1} V(t) = v_1$$

Similarly,

$$\lim_{p \to \widehat{p_1}} \widehat{P^{-1}}(p) = \widehat{p_1}$$

and

$$\lim_{p \to p_1} P^{-1}(p) = p_1$$

Thus

$$\lim_{p \to \widehat{p_1}} (\widehat{V}(\widehat{P^{-1}}(p)) - V(P^{-1}(p))) = \widehat{v_1} - v_1 \neq 0$$

So there exists some $q \in (p_0, p_1)$ such that

$$\widehat{V}(\widehat{P}^{-1}(q)) = V(P_{-1}(q))$$

□

Thus there is no function, P, that can act as a perspective such that $V||^{P}_{(0,1)}$. This really shouldn't be very surprising. After all, the block really does move faster. The only reason that the period is unchanged is that the increased velocity is exactly counterbalanced by the increased distance the block must travel. It would be foolish to try and claim the velocity doesn't increase when it does. Instead, an intuitive explanation should account for the balance of the change in velocity and distance. This type of explanation is outside the realm of DQ analysis, and probably beyond the abilities of qualitative physics as well.

3.10 Extensions for Diagnosis

A natural application for comparative analysis is the automated diagnosis of continuous devices such as analog electronic circuits. Generate and test is a standard paradigm for hardware diagnosis [9]: candidate faults are proposed then evaluated to see if they account for the faulty measurements. Like all forms of comparative analysis, DQ analysis can be used to test any candidate faults that can be described as perturbations of continuous parameters in the device. In addition, however, DQ analysis has the potential to generate classes of candidate faults. The key is to run the DQ inference rules in reverse. For example, the duration rule says:

Proposition 1 Duration Rule
*Let V and X be parameters such that X is a partial perspective over (γ_i, γ_{i+1}). Given $V = \frac{d}{dt}X$, $V{\Downarrow}^{X}_{(i,i+1)}$, and $\neg$*DISTANCE-BY*$X{\Downarrow}_{(i,i+1)}$ then the duration of (γ_i, γ_{i+1}) will increase.*

A natural question is "Can the duration rule be reversed? Is the converse sound?"

Conjecture 14 Converse Duration Rule
*Let V and X be parameters. Given $V = \frac{d}{dt}X$ and $\neg$*DISTANCE-BY*$X{\Downarrow}_{(i,i+1)}$. If the duration of (γ_i, γ_{i+1}) $\Uparrow$, then $V{\Downarrow}^{X}_{(i,i+1)}$.*

Unfortunately, the converse is false, as are the converses for other important rules such as the various derivative rules. The problem results from an implicit closed world assumption used in reversing the rule—that one of the three RC values, $\Uparrow$, $\Downarrow$, or $\|$, always applies. Proposition 12 showed that this was false.

Of course every transition interval could be broken into pieces such that a single RC value applies over each piece, but this misses the fundamental issue. The decomposition of time into transition intervals is forced by behavior of the system. Thus transition intervals have genuine qualitative importance. While sometimes useful, decomposing transition intervals into smaller pieces runs the risk of introducing irrelevant distinctions.

Although the converse of the duration rule is not sound, its converse might still be profitably used as a heuristic candidate generator. By reversing the DQ inference rules, it may be possible to provide focus to the search for probably faults in misbehaving analog circuitry.

4 Changes in Behavioral Topology

Recall that the inference rules of section 3 relied on the assumption that the perturbed behavior was topologically equal (definition 8) to the initial behavior. In other words, it was assumed that while the perturbation might change the relative values of parameters and stretch or shrink the length of time intervals, the underlying sequence of transitions would not change. Yet perturbations often will change the order or nature of transitions. This section explains how to recognize the changes and predict the resulting behavior.

To illustrate these computations, I use the simple example of the heat exchanger (figure 4) from section 1.2. This system is described in terms of five parameters, each a function of time: heat Q, heat flow F, thermal conductivity $-K$, velocity of the liquid through the pipe V, and position of a unit volume of oil[6] X. The following equations are obeyed:[7]

$$\begin{aligned} V &= \tfrac{d}{dt}X \\ F &= \tfrac{d}{dt}Q \\ F &= \textsc{mult}(Q, -K) \end{aligned} \tag{4}$$

In addition V and $-K$ are considered independent and assumed constant over time. The initial conditions specify the value for the independent parameters: $V(0) > 0$ and $-K(0) < 0$, and also the boundary conditions: $X(0) = x_0 < 0$ and $Q(0) = q_0 > 0$. From this information the initial value of the dependent parameter, F, can be determined; denote $F(0) = f_0$. An invariant specifying that X must always be less than or equal to zero ends the simulation when the liquid individual leaves the pipe.

Given this description, QSIM (and other qualitative simulators [6]) produces the tree of qualitative states (STATE TREE) shown in figure 11. Since each path through the tree is a topologically distinct behavior, this tree represents three possible behaviors for the heat exchanger. The topmost path (QS1, QS2, QS3) corresponds to the behavior of figure 12 in which the system reaches thermal equilibrium just as the oil leaves the exchanger.

Because of its qualitative representations, QSIM cannot choose between the different behaviors for the heat exchanger; as far as QSIM is concerned, they are all plausible. Since DQ analysis works relative to a single behavior, one path through the tree must be chosen before running the rules of section 3. This selection of a behavior is a modeling decision; I assume that it is done by a human. The selection consists of a series of choices at each branch in the tree. By ruling out possible behaviors, each choice implicitly constrains the model of the system, restricting the possible real values associated with the qualitative values of each parameter. Thus the selection of behavior (QS1, QS2, QS3)

[6] For simplicity, the simplistic 'liquid-individual' model of fluids is used here; see [10] for a discussion of the problems with this model.

[7] For simplicity, this model does not distinguish between temperature and heat.

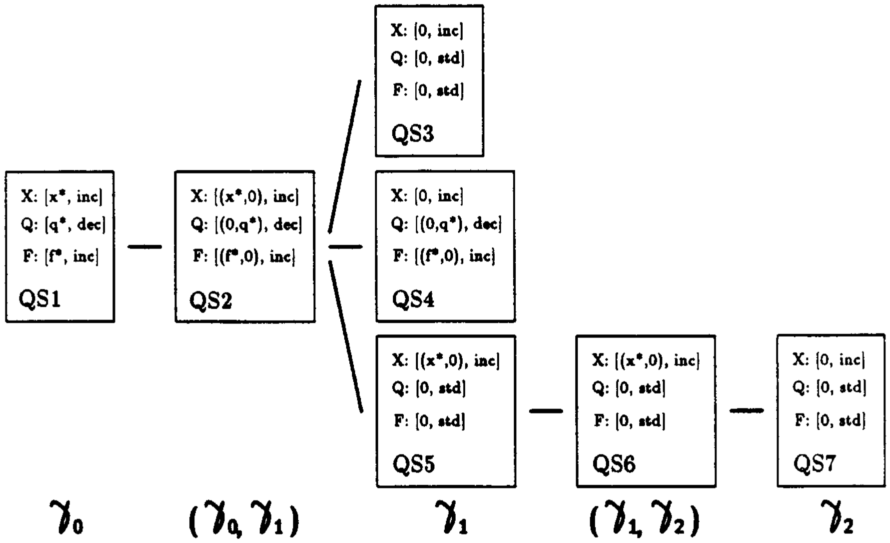

Figure 11: QSIM State Tree Generates Possible Behaviors

makes implicit assumptions about the relative values of fluid velocity, V, and thermal conductivity, $-K$.

These implicit constraints are equivalent to the unambiguous selection of the initial behavior. However, the comparative analysis perturbation can weaken the balance of constraint in two ways:

- The initial behavior can be rendered inconsistent. Section 4.1 explains how the conflict is recognized and a new, consistent path is found.
- Alternate behaviors may become consistent. Section 4.2 explains how to locate other consistent paths through the state tree.

4.1 Initial Behavior Inconsistent

Suppose someone selected the path (QS1, QS2, QS3) as the heat exchanger's initial behavior (figure 12) and chose the perturbation $-K\Uparrow$. The state QS3 dictates the two transitions, Q reaching zero and X reaching zero, in the same time instant. Since the perturbation causes heat to be lost more rapidly, QS3 can't be part of the final behavior. If one assumes that it is, the duration rule (section 3.1) deduces a contradiction, as follows.

When $-K\Uparrow$, it follows that $F\Uparrow^{Q}_{(0,1)}$.[8] Thus the duration until the first transition is $\Downarrow$. However, being a constant V is unchanged by the perturbation, so

[8] By the interval constant rule, the self reference rule and the multiplication rule.

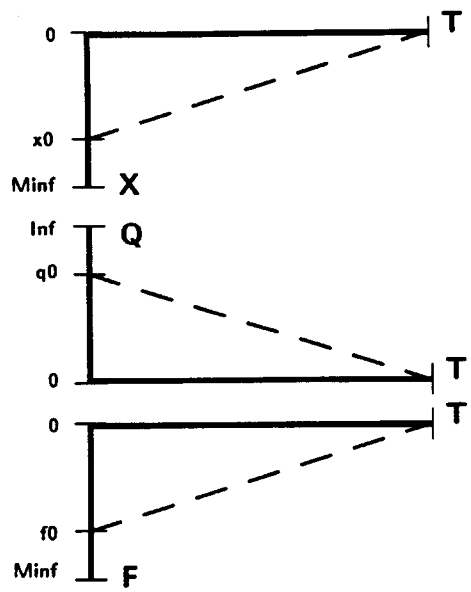

Figure 12: Initial Behavior Corresponding to Path (QS1, QS2, QS3)

$V\|^{X}_{(0,1)}$; the duration rule uses this fact to conclude that the duration $\|$. Hence the conflict. The perturbation causes heat to reach its transition quicker, but position is unaffected and will transition at the same time.

Behavioral inconsistencies are located by stepping through the transition intervals from earliest on, and checking the RC values for the interval's duration. Section 4.1.1 explains how to find all behaviors that avoid this single contradiction while obeying the initial constraints. Section 4.1.2 provides heuristics for eliminating inferior paths. Finally, section 4.1.3 shows how to check if the new behavior is globally consistent, not just a fix to the first contradiction. Note that all of these techniques depend on the DQ inference rules which are incomplete. As a result, while most inconsistencies are detected, it is not guaranteed that all inconsistencies can be found.

4.1.1 Finding Consistent Alternatives

A simple observation about the inference rules of section 3 forms the foundation for the contradiction resolution method: only the duration rule can generate an RC value for a time duration. Therefore, the contradiction must be caused by two (or more) firings of the duration rule for the same interval. What distinguishes these firings are the different perspective parameters used in each application of the rule.

In the heat exchanger example, the two perspective parameters are Q and

X. In the initial behavior, they reached transitions in the same state, QS3. But to achieve consistency with the perturbation, we must find a behavior where they reach transitions independently. This means finding a path which starts with QS1 and QS2, and passes through a sibling of QS3. The answer, of course, is the path (QS1, QS2, QS5, QS6, QS7) as shown in figure 13. This path illustrates the general case. A node representing the qualitative state at a time point (QS3) is replaced by three states: two at time points (QS5, QS7) and one for the interval connecting them (QS6). For the purpose of discussion, I shall call QS3 the FRAGMENTING POINT and the two time-point states which define our objective, the PREPOINT and POSTPOINT respectively.[9]

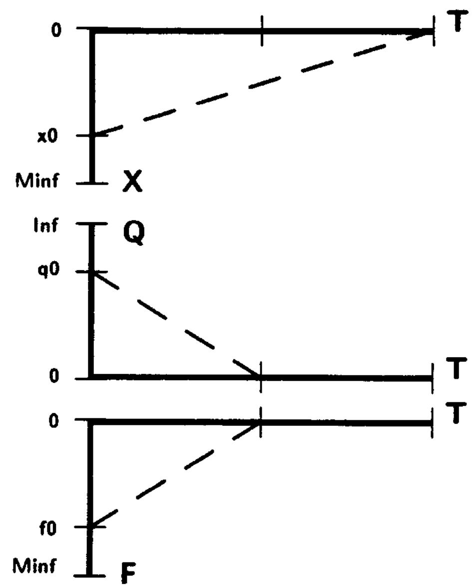

Figure 13: The Behavior Corresponding to Path (QS1, QS2, QS5, QS6, QS7)

The problem, then, is to search the state tree among the siblings of the fragmenting point to find the pre- and postpoint states. We know that Q and X must reach transitions in different states, but which should reach its transition first? Consider the two duration RC values which cause the contradiction. Since $\Downarrow$ specifies earlier termination than $||$, Q, the perspective parameter for the firing which produced the $\Downarrow$ value, will reach its transition first. This means that the

[9] Actually, this discussion assumes a simplified version of the general problem. I assume that the contradiction is caused by only two firings of the duration rule, and I assume that at the contradiction can be resolved by the addition of a single new transition. The general case is a straightforward extension. If the QS2 interval had three conflicting duration RC values, $\Downarrow$, $||$, and $\Downarrow$, then QS3 could split into five states: three for time points and two connecting intervals. If multiple rule firings are allowed for each RC value, then correspondingly more paths are possible.

prepoint will have $Q = 0$ and not $X = 0$. Since QS5 is the only state to meet this requirement, so QS5 is the prepoint. For the heat exchanger example, this state uniquely defines the new behavior, because only one path includes QS5; hence QS7 must be the postpoint. In a more complex example, however, there could be several candidates for prepoint and multiple behaviors passing through each one. The following conditions further restrict the possibilities.

- All parameters that reach transitions in the prepoint, must have reached transitions in the fragmenting point.
- All parameters that reach transitions in the postpoint, must have reached transitions in the fragmenting point.
- All parameters that reached transitions in the fragmenting point must reach transitions in either the prepoint or the postpoint, but not both.

While these conditions are loyal to the implicit constraints resulting from the initial selection of behavior, they are unfortunately not sufficient to guarantee a unique alternate behavior. The next section explains a heuristic that will guarantee a unique behavior but not necessarily one that obeys all implicit constraints.

4.1.2 A Heuristic For Eliminating Behaviors

The conditions listed above produce a unique behavior except in cases where additional parameters besides Q and X reach transitions in the fragmenting point. When extra parameters reach transitions in the fragmenting point, one must choose where they should transition—in the prepoint or the postpoint. The following cases result:

- The parameter could be causally connected to either Q or X. In fact, this is the case with the heat exchanger: F transitions to zero in QS3. How did we know that F should reach its transition in the prepoint rather than the postpoint? We didn't even need to consider the question. By constructing the state tree, QSIM already handled the problem for use. It recognized that F must transition whenever Q transitioned; thus the state tree contains only this possibility. Since the topological consistency code searches the state tree, it automatically benefits from QSIM's work.
- There could be additional RC information about the parameter. For simplicity, this case was not discussed above, but suppose that the duration rule had fired three times with Q, X, and S as perspectives. If S and X both caused the duration rule to deduce an RC value of $||$, then both X and S should reach transitions in the postpoint. Unfortunately, other RC values complicate the analysis. If three different RC values result from the three firings, then the fragmenting point will split into five states.

The details are messy, but the concepts for resolution are similar to those described above.

- The parameter could be independent of the perturbation without the inference rules deducing this. As with the previous case (where the independence, duration ||, *was* deduced) the parameter should transition in whichever state has duration ||.

- The perturbation could change the parameter's transition time without the inference rules deducing this. The correct behavior is not predictable since the change in duration is not known.

Since there is no way to correctly handle the last case, a reasonable heuristic is to assume that it never happens. This corresponds to Occam's Razor. Assume that unless the duration rule says otherwise, the perturbation does not change the transition time of any parameters. Thus if the heat exchanger example had an extra parameter, S, which reached a transition in QS3, then we should assume that S transitions with X in QS7.

4.1.3 Ensuring Global Consistency

Using the heuristics, the algorithm described above is guaranteed to find a unique postpoint. But there may be several state tree paths that pass through this post point. To locate a single new behavior, the program must step through the original behavior from the fragmenting point onwards. Every time a branch in the tree is taken, the corresponding descendant of the postpoint should be selected as well. When the original behavior reaches a leaf, a unique new behavior will result. Unfortunately, there are two reasons why processing must continue.

- Many RC values must be recomputed. Because the RC values refer to transition points and intervals, all values from the fragmenting interval onward will be incorrect. This isn't very surprising; after all, we started with conflicting duration RC values in the first place. Given the new behavior, the inference rules of section 3 must be rerun to generate a consistent set of RC values.

- What if these rules generate a new contradiction? There is no guarantee that the new behavior is topologically sound. However, if conflicting duration RC values *are* generated for an interval, that interval must occur after any interval which caused a previous conflict. Thus each cycle of inference rules and topology resolution guarantees that the time of first inconsistency increases. Since all behaviors are finite, the cycle must eventually terminate.

It is possible that a more sophisticated algorithm could eliminate this cyclic approach by a detailed analysis of the initial behavior. Since all parameters are continuous functions of time, no (small) perturbation can invalidate the initial behavior unless the behavior has a state in which two parameters transition. Perhaps all such states could be checked at once.

4.2 Finding Other Consistent Behaviors

Sometimes a perturbation will be consistent with the initial behavior, i.e. not violate the implicit constraints, but will weaken them instead. In other words, there may be several behaviors which are consistent with the perturbed initial behavior. Since the QSIM state tree records the results of past transition analysis, a simple search technique suffices to find the behaviors that are consistent with both the perturbation and the implicit constraints. Four cases need to be checked: compacting, stalling, kick-starting, and splitting.

- COMPACTING

  When the duration of an interval is decreasing, perhaps the states on either side will merge into a single transition. Suppose the initial behavior is the path (QS1, QS2, QS5, QS6, QS7) as shown in 13, and suppose the perturbation is $-K\Downarrow$. Although the initial behavior is consistent with this perturbation, it is possible that thermal equilibrium will be delayed until the precise moment that the oil leaves the pipe. This would correspond to the behavior (QS1, QS2, QS3) as shown in figure 12. Whenever the duration of an interval (e.g., QS6) is getting shorter, CA looks for an uncle state which has the same transitions (i.e. the same parameters reaching the same landmarks) as the union of the parent and child of the interval state. Q and F reach transitions in QS5, the parent of QS6, and X transitions in the child of QS6. So the search produces the uncle, QS3, and constructs the corresponding path through it.

- STALLING

  If the duration of an interval is $\Uparrow$, then maybe the parameters will not transition in finite time. CA suggests a behavior consisting of the path which ends at the interval state.

- KICK-STARTING

  Kick starting is the inverse of stalling. If the initial behavior ended with a terminal interval, and the perturbation is causing the interval's duration to $\Downarrow$, then maybe some parameter will transition in finite time. CA returns all paths that pass through the interval state.

- SPLITTING

  Splitting happens when the duration rule[10] deduces a single RC value of $\Uparrow$ or $\Downarrow$ from two different perspectives. For example, consider the heat exchange with an initial behavior of (QS1, QS2, QS3) and the perturbation of $-K\Uparrow$ and $V\Uparrow$. Since thermal conductivity is higher, equilibrium will occur sooner, but since the oil is moving faster, it will get out quicker. Both parameters lead the duration rule to conclude QS3 will occur quicker; thus the initial behavior is consistent. But so is every behavior. If the thermal conductivity is much higher and velocity is only a little higher, then the behavior (QS1, QS2, QS5, QS6, QS7) will result. If V was increased more than $-K$ then the path (QS1, QS2, QS4) would result. Because the perturbation was specified in qualitative terms, there isn't enough information to resolve the ambiguity and CA must return all possible splits of the two parameters Q and X.

Like the techniques of section 4.1, my methods for finding other consistent behaviors are dependent on the DQ inference rules. As a result they are neither complete nor sound. For example, suppose the duration of an interval was $\Downarrow$, but the duration rule had not deduced this fact. Then compaction would not be considered and a possibly consistent behavior would not be considered. Similarly, one of the techniques could suggest a behavior which appears consistent only because the DQ rules were inadequate to expose a contradiction.

[10] Splitting is the only case that analyzes justifications and depends on the fact that the duration rule is the only way to generate a duration RC value. Compacting, stalling, and kick-starting only require the RC value and access to the state tree.

5 Related Work

Although comparative analysis questions have long been important topics in the fields of engineering and mathematics, little work has been done on comparative analysis in the artificial intelligence community.

5.1 Sensitivity Analysis

Sensitivity analysis is a common engineering technique for calculating the effect on system performance due to variations in system parameters. In other words, comparative analysis is a qualitative version of sensitivity analysis. The sensitivity of a quantity T, to perturbations in a parameter X, is defined [1] as the product:

$$\frac{\partial T}{\partial X}\frac{X}{T}$$

Because of its important application to design, considerable work has been done on efficient methods for calculating sensitivities. Approaches include numerical and symbolic differentiation, construction of an incremental network, and analysis of an adjoint network [1].

Compared to DQ analysis, these methods have a major advantage—they generate a quantitative value for sensitivity. But sensitivity analysis has two limitations: it does not generate explanations, and it requires an explicit equation for the desired quantity T. Thus sensitivity analysis could not solve the spring/block problem until the human modeler provided a formula for period.

The technique of comparative statics [17,12], long used in economics to compare two different equilibrium behaviors, suffers from the same limitation. It requires explicit formulas for the partial derivatives in question.

5.2 Partial Derivatives

Since the RC notation expresses how a parameter changes given an initial perturbation, it is natural to ask about its relationship to the standard mathematical tools for expressing relative change: partial derivatives. In the following proposition it is handy to think of parameter C as the cause, and E as an effect.

Proposition 15 *If $C\Uparrow_0$ and all other independent and boundary condition parameters have an RC value of $||_0$ and $E\Downarrow_0$ then*

$$\frac{\partial E}{\partial C} < 0$$

at time zero.

This statement can be extended to any transition, γ_i, by normalizing with respect to time. While the relationship between RC values and partial derivative is straightforward for values at transition points, the connection is more subtle for interval RC values because of the presence of perspectives.

5.3 QP Theory

In his treatise on Qualitative Process theory, Forbus discussed differential qualitative analysis [7, pages 159-161], but attempted no implementation. He defined quantities q_1 greater than q_2 over an interval, i, if for all instants in the interval, $q_1 > q_2$ measured at that instant. Unfortunately, this definition has several problems. Since the quantification is over a single interval of time, it is impossible to make comparisons of systems whose time behavior changes as a result of a perturbation. Thus his attempt to formalize "distance equals rate times duration" in predicate calculus is severely limited. Rates can only be compared if the duration of an interval is unchanged!

But even if the quantification was correct, time-wise comparison is almost never a useful one to make. In the spring/block case, for example, it simply isn't the case that the heavy-block is always moving slower than the small-block; the periods get out of phase. The key to solving these problems is in the use of perspectives, discussed in this paper. The comparison on velocity (necessary to predict that the period lengthens) is valid only from the perspective of position.

5.4 Temporal Representation

QSIM [15] is an efficient, easy to use simulator that has significantly sped the development of both my comparative analysis theory and the CA implementation. However, QSIM has defects; its weak temporal representation is a major problem.

As explained by Hayes in [11], systems which represent behaviors as a sequence of states force a total ordering on events. Because qualitative reasoning is often unable to unambiguously determine an order, the behavior must branch to consider multiple possibilities. If events interact, then the various branches often have interesting qualitative differences. But frequently, the alternate behaviors are equivalent and just complicate reasoning and consume processing resources.

To combat this problem, Williams introduced the notion of concise episodes [23], and has devised an efficient simulator (called a Temporal Constraint Propagator) to manipulate them. Just as qualitative simulators using Williams' temporal representation would improve on QSIM, comparative analysis programs would have several advantages over CA. Williams is building such a system for use in automated design [25].

- The propositions of section 3 would still be true, and could be encoded more easily. CA requires explicit rules for composing durations over intervals (e.g., if $\text{DURATION}\Uparrow_{(0,1)}$ and $\text{DURATION}\Uparrow_{(1,2)}$ then $\text{DURATION}\Uparrow_{(0,2)}$). These computationally expensive rules would be subsumed by the temporal constraint propagator.

- The search for topologically distinct behaviors (section [illegible]) would be simplified because the space would be smaller. By eliminating irrelevant order distinctions, the number of different behaviors would be smaller. Only if it was qualitatively interesting would there be any need to consider a behavior in which two parameters [illegible] simultaneously.

6 Future Directions

This paper discussed the problem of comparative analysis, the task of explaining how a system will react to perturbations, and why. Multiple perspectives, which can be used to reparameterize system parameters, lead to a powerful definition of relative change. DQ analysis solves comparative analysis problems by applying inference rules to the initial perturbation of a system. A trace of the rules used in solving a problem can be easily translated into an intuitive explanation of the answer. Since the rules have been proven sound, DQ analysis is guaranteed to produce only correct explanations. A computer programs, CA, implements the theory of DQ analysis and correctly solves over twenty comparative analysis problems including those that change the order of transitions in the behavior.

Despite the success of DQ analysis, several areas for future research beckon.

- DQ analysis is incomplete. Although CA is guaranteed to terminate, it doesn't always deduce an RC value. Fortunately, there are other techniques for solving comparative analysis problems. Exaggeration, for example, saves many problems with a completely different style of reasoning [20]. Although it is believed that exaggeration is also incomplete, initial results suggest that exaggeration can solve several problems which DQ analysis cannot [22].

- Certain comparative analysis questions have no answer. For example, "What happens to the period of oscillation of a spring/block if both the mass and spring constant are increased?" It would be nice if CA could recognize that there was no answer to this question instead of simply saying that it can't find an answer.

- Any analytic technique is only as good as the model on which it works. Currently, humans construct models and computers are only used in analyzing them. This imparts fragility to the process. For example, consider the structural description of the oscillating spring/block example (section 3.8). Suppose that the initial situation had $X = 0$, V set to some maximum value and the perturbation was $V\Uparrow$. Although the DQ inference rules can deduce facts like "the maximum displacement will increase," the topological analyzer is unable to recognize the possibility that the spring will break.

  The cause is a simplistic model. Hooke's law precludes the possibility of a broken spring. To achieve greater robustness in qualitative analysis in general and comparative analysis in specific, modeling must be treated explicitly. By incorporating ontological assumptions into process definitions, QP theory [7] has made progress here, but further research is necessary to address the questions of reasoning with multiple models, dynamic model creation, and the evaluation of a model in the context of a specific problem.

A A Useful Example

This section constructs an example which serves both as a counter-example for a generalized version of the derivative rule and as the proof of the non-exhaustive proposition. Suppose that A, V, and X are parameters such that $A = \frac{d}{dt}V$, $V = \frac{d}{dt}X$, and X is a covering perspective over (γ_i, γ_{i+1}). The derivative rule (proposition 3) showed that if A and V are positive over the interval (γ_i, γ_{i+1}) and if $\neg V{\Uparrow}_i$, and $A{\Downarrow}^X_{(i,i+1)}$, then $V{\Downarrow}^X_{(i,i+1)}$.

Unfortunately, the derivative rule is not true for arbitrary perspectives. The following abberation should convince you of this. I show three parameters, V, A, and P such that $A = \frac{d}{dt}V$ and P is a covering perspective over (γ_0, γ_1). Yet although $A||^P_{(0,1)}$ the parameter V has no consistent behavior from the perspective of P. During part of the interval $V{\Uparrow}^P$ and during part $V{\Downarrow}^P$.

Here are the details. Over the absolute time interval $(0, 1)$ define:

$$
\begin{array}{ll}
V(t) = \frac{1}{2}t^2 & \\
A(t) = \frac{d}{dt}V(t) = t & A^{-1}(a) = a \\
P(t) = A(t) = t & P^{-1}(p) = p \\
\widehat{V}(t) = \frac{1}{3}t^3 & \\
\widehat{A}(t) = \frac{d}{dt}\widehat{V}(t) = t^2 & \widehat{A}^{-1}(a) = a^{\frac{1}{2}} \\
\widehat{P}(t) = \widehat{A}(t) = t^2 & \widehat{P}^{-1}(p) = p^{\frac{1}{2}}
\end{array}
$$

Note that $P(0) = \widehat{P}(0) = 0$ and $P(1) = \widehat{P}(1) = 1$ and P is strictly monotonic, so P is a valid perspective over this interval. Since $P = A$ and $\widehat{P} = \widehat{A}$ the self reference theorem shows that $A||^P_{(0,1)}$. So what does V do from the perspective of P? Consider $p = \frac{1}{9}$:

$$V(P^{-1}(p)) = \frac{1}{2}p^2 = \frac{1}{162}, \text{ and } \widehat{V}(\widehat{P}^{-1}(p)) = \frac{1}{3}p^{\frac{3}{2}} = \frac{1}{81}$$

Now let $p = \frac{8}{9}$:

$$V(P^{-1}(p)) = \frac{1}{2}p^2 = \frac{32}{81}, \text{ and } \widehat{V}(\widehat{P}^{-1}(p)) = \frac{1}{3}p^{\frac{3}{2}} = \frac{16\sqrt{2}}{81} < \frac{32}{81}$$

So for a small value of p the corresponding $\widehat{v}$ is larger than v, but for larger p the situation is reversed. Thus it is neither the case that $V{\Uparrow}^P_{(0,1)}$ nor $V||^P_{(0,1)}$ nor $V{\Downarrow}^P_{(0,1)}$ even though $A||^P_{(0,1)}$.

References

[1] L. Chua and P. Lin. *Computer Aided Analysis of Electronic Circuits: Algorithms and Techniques.* Prentice Hall, Englewood Cliffs, NJ, 1975.

[2] J. de Kleer and J. Brown. A Qualitative Physics Based on Confluences. *Artificial Intelligence*, 24, December 1984.

[3] J. de Kleer, J. Doyle, G. L. Steele, and G. J. Sussman. AMORD: Explicit Control of Reasoning. In *Proceedings of the Symposium on Artificial Intelligence and Programming Languages*, SIGART, Rochester, N.Y., August 1977.

[4] Williams B. de Kleer, J. Back to Backtracking: Controlling the ATMS. In *Procceedings of the AAAI*, August 1986.

[5] Gerald DeJong and Raymond Mooney. Explanation-based learning: an alternative view. *Machine Learning*, 1(2), 1986.

[6] K. Forbus. The logic of occurrence. In *Proceedings of the Tenth IJCAI*, pages 409–415, August 1987.

[7] K. Forbus. Qualitative Process Theory. *Artificial Intelligence*, 24, December 1984.

[8] K. Forbus and A. Stevens. *Using Qualitative Simulation to Generate Explanations.* Technical Report 4490, Bolt Beranek and Newman Inc, March 1981.

[9] W. Hamscher and R. Davis. *Issues in Diagnosis from First Principles.* AI Memo 893, MIT AI Lab, March 1987.

[10] P. Hayes. Naive Physics I: Ontology for Liquids. In *Formal Theories of the Commonsense World*, Ablex, 1985.

[11] P. Hayes. The Second Naive Physics Manifesto. In *Formal Theories of the Commonsense World*, Ablex, 1985.

[12] Y. Iwasaki and H. Simon. Causality In Device Dehavior. *Artificial Intelligence*, 29(1):3–32, July 1986.

[13] S. T. Kedar-Cabelli and L. T. McCarthy. Explanation-based generalization as resolution theorem proving. In *Proceedings of the Fourth International Workshop on Machine Learning*, June 1987.

[14] B. Kuipers. The limits of qualitative simulation. In *Proceedings of the Ninth IJCAI*, August 1985.

[15] B. Kuipers. Qualitative Simulation. *Artificial Intelligence*, 29, September 1986.

[16] Tom M. Mitchell, Richard M. Keller, and Smadar T. Kedar-Cabelli. Explanation-based generalization: a unifying view. *Machine Learning*, 1(1), 1986.

[17] P. Samuelson. *Foundations of Economic Analysis.* Harvard University Press, Cambridge, MA, 1947.

[18] P. Struss. Mathematical aspects of qualitative reasoning. *International Journal for Artificial Intelligence in Engineering*, To Appear 1988.

[19] D. Weld. *Explaining Complex Engineered Devices.* Technical Report 5489, Bolt Beranek and Newman Inc, November 1983.

[20] D. Weld. Preliminary report on exaggeration. *International Journal for Artificial Intelligence in Engineering*, To Appear 1988.

[21] D. Weld. The Use of Aggregation in Qualitative Simulation. *Artificial Intelligence*, 30(1), October 1986.

[22] D. Weld. *The Theory and Evaluation of Comparative Analysis Techniques.* PhD Thesis, MIT Artificial Intelligence Lab, To Appear 1988.

[23] B. Williams. Doing Time: Putting Qualitative Reasoning on Firmer Ground. In *Proceedings of the National Conference on Artificial Intelligence*, pages 105–112, August 1986.

[24] B. Williams. *Exorcising Control of Demon Invocation: A Study in Tractable Non-Monotonicity.* AI Memo 944, MIT Artificial Inteligence Lab, 1987.

[25] B. Williams. *Principled Design Based on Qualitative Behavioral Descriptions.* PhD Thesis Proposal, MIT Artifical Intelligence Lab, November 1985.

[26] B. Williams. Qualitative Analysis of MOS Circuits. *Artificial Intelligence*, December 1984.

CS-TR Scanning Project
Document Control Form

Date : 10 / 12 / 95

Report # AIM-951

Each of the following should be identified by a checkmark:
Originating Department:

- [x] Artificial Intellegence Laboratory (AI)
- [ ] Laboratory for Computer Science (LCS)

Document Type:

- [ ] Technical Report (TR)
- [x] Technical Memo (TM)
- [ ] Other: ____________

Document Information

Number of pages: 47 (53-images)
Not to include DOD forms, printer intstructions, etc... original pages only.

Originals are:

- [x] Single-sided or
- [ ] Double-sided

Intended to be printed as :

- [ ] Single-sided or
- [x] Double-sided

Print type:

- [ ] Typewriter
- [ ] Offset Press
- [ ] Laser Print
- [ ] InkJet Printer
- [ ] Unknown
- [ ] Other: ____________

Check each if included with document:

- [x] DOD Form (2)
- [ ] Funding Agent Form
- [ ] Cover Page
- [ ] Spine
- [ ] Printers Notes
- [ ] Photo negatives
- [ ] Other: ____________

Page Data:

Blank Pages(by page number): ____________

Photographs/Tonal Material (by page number): ____________

Other (note description/page number):

Description : Page Number:

IMAGE MAP (1-47) UN#'ED TITLE PAGE, 1, 1-43, NO PAGE 44, 45-46 (BLANK?)
(48-53) SCANCONTROL, DOD(2), TRGT'S (3)

Scanning Agent Signoff:

Date Received: 10 / 12 / 95 Date Scanned: 10 / 18 / 95 Date Returned: 10 / 19 / 95

Scanning Agent Signature: Michael W. Cook

Rev 9/94 DS/LCS Document Control Form cstrform.vsd

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 951	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER AD-A190556	
4. TITLE (and Subtitle) Comparative Analysis	5. TYPE OF REPORT & PERIOD COVERED AI Memo
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Daniel S. Weld	8. CONTRACT OR GRANT NUMBER(s) ONR N00014-85-K-0124
9. PERFORMING ORGANIZATION NAME AND ADDRESS Artificial Intelligence Laboratory 545 Technology Square Cambridge, MA 02139	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Advanced Research Projects Agency 1400 Wilson Blvd. Arlington, VA 22209	12. REPORT DATE November 29, 1987 13. NUMBER OF PAGES 46
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research Information Systems Arlington, VA 22217	15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Comparative Analysis, Qualitative Physics, Qualitative Simulation, DQ Analysis, QSIM, Perspectives, Explanations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Comparative analysis is the problem of predicting how a system will react to perturbations in its parameters, and why. For example, comparative analysis could be asked to explain why the period of an oscillating spring/block system would increase if the mass of the block were larger. This paper formalizes the problem of comparative analysis and presents a technique, differential qualita-

(over)

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Completed 7-28-88

Block 20 Continued:

tive (DQ) analysis, which solves the task, providing explanations suitable for use by design systems, automated diagnosis, intelligent tutoring systems, and explanation based generalization.

DQ analysis uses inference rules to deduce qualitative information about the relative change of system parameters. Multiple perspectives are used to represent relative change values over intervals of time. Differential analysis has been implemented, tested on a dozen examples, and proven sound. Unfortunately, the technique is incomplete; it always terminates, but does not always return an answer.