Division 6 - Lincoln Laboratory
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Subject: TRANSISTOR CIRCUITS COURSE
NUMBER 2. EQUIVALENT CIRCUITS OF TRANSISTORS

To: Distribution List

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Approved: DRB
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Abstract: A number of different equivalent circuits are used to represent the transistor and an understanding of these is imperative to the circuit designer. The open-circuit impedance representation, the equivalent-T representation, and the hybrid representation are all commonly used. An important modification of the standard T-circuit was made by J. M. Early. A π-variation of this has proven useful for surface-barrier transistors.

1.0 Open-Circuit Impedance Representation of a Transistor

Suppose we consider the transistor as a four-terminal device as shown below in Fig. 1. The transistor can be represented as a

a.)

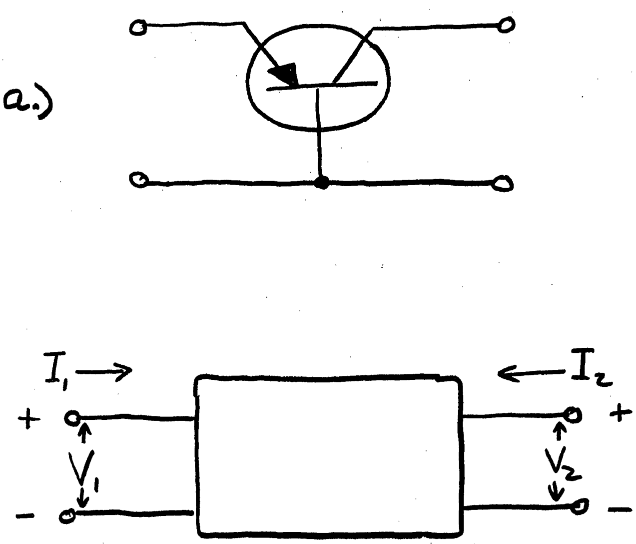


FIG. 1

The research reported in this document was supported jointly by the Department of the Army, the Department of the Navy, and the Department of the Air Force under Air Force Contract No. AF 19(122)-458.

"black box" with input and output currents and voltages as specified. A general representation of such a black box is given in Fig. 2 with the circuit equations written below.

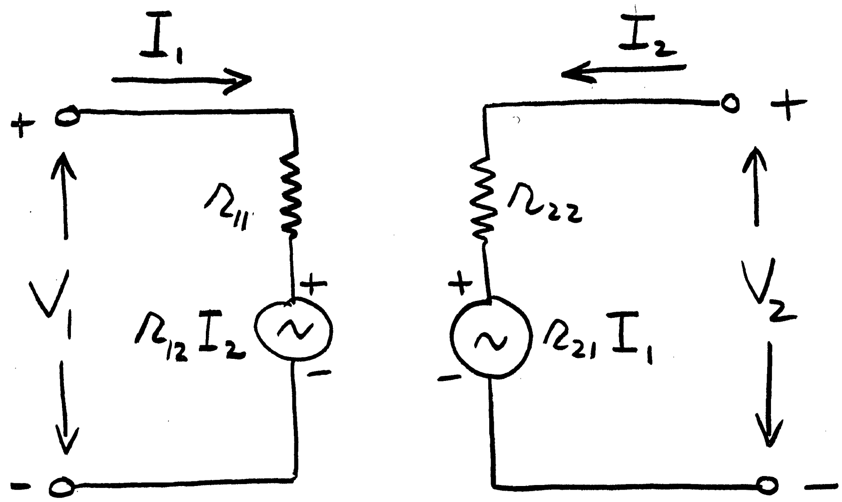


FIG. 2 - FOUR TERMINAL NETWORK

$$V_1 = r_{11}I_1 + r_{12}I_2 \quad (1)$$

$$V_2 = r_{21}I_1 + r_{22}I_2 \quad (2)$$

We can express the coefficients of the currents by the following derivatives:

$$\left[\frac{\partial V_1}{\partial I_1}\right]_{I_2\ \text{CONST}} = r_{11}. \qquad \left[\frac{\partial V_2}{\partial I_1}\right]_{I_2\ \text{CONST}} = r_{21}.$$

$$\left[\frac{\partial V_1}{\partial I_2}\right]_{I_1\ \text{CONST}} = r_{12}. \qquad \left[\frac{\partial V_2}{\partial I_2}\right]_{I_1\ \text{CONST}} = r_{22}.$$

These coefficients r_{ij} are called the open-circuit impedances of the transistor since they are the relations between current and voltage (i.e. impedance) in equations (1) and (2) if one current is made zero (i.e. open-circuited). If these impedances are known, then equations (1) and (2) specify the operation of the transistor and Fig. 2 is an equivalent circuit. There are, however, other more useful representations.

2.0 Equivalent-T Representation

Perhaps the most common and generally useful representation of a transistor is the T-equivalent circuit. This is largely because it presents a picture readily related to the actual physical construction of the transistor. The equivalent-T circuit is drawn in Fig. 3. Here the subscripts c and e are used for collector and emitter. Equations (3) and (4) represent the circuit.

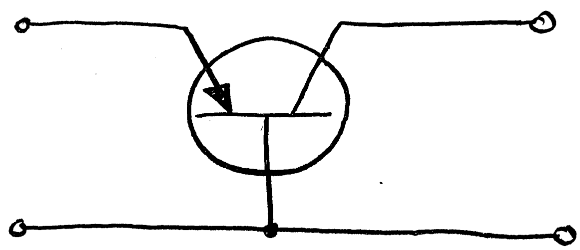
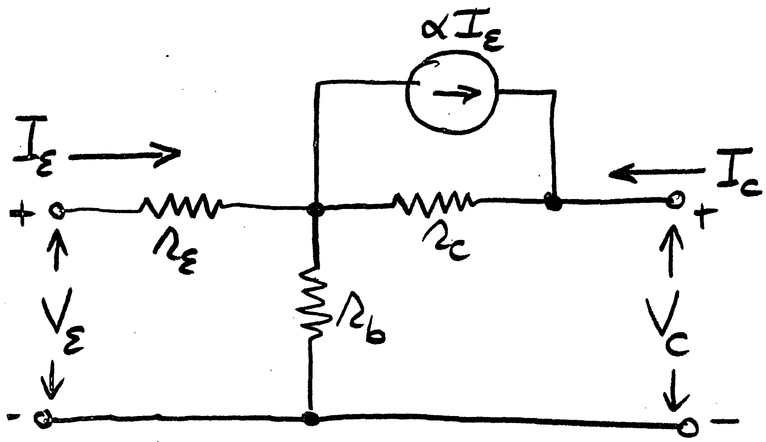


FIG. 2



$$V_e = (r_e + r_b)I_e + r_b I_c \qquad (3)$$

$$V_c = (a r_c + r_b)I_e + (r_c + r_b)I_c \qquad (4)$$

If we now rewrite (1) and (2) with new subscripts we get:

$$V_e = r_{11} I_e + r_{12} I_c \quad (1a)$$

$$V_c = r_{21} I_e + r_{22} I_c \quad (2a)$$

Comparing the two sets of equations above gives the following relations between the equivalent-T impedances and the open-circuit impedances:

$$r_{11} = r_e + r_b$$

$$r_{12} = r_b$$

$$r_{21} = a r_c + r_b$$

$$r_{22} = r_c + r_b$$

These quantities can be expressed by the derivatives given before and, in particular, the base and collector resistances are given by:

$$r_c \approx r_{22} = \left| \frac{\partial V_c}{\partial I_c} \right| I_e \text{ const}$$

$$r_b = r_{12} = \left| \frac{\partial V_e}{\partial I_c} \right| I_e \text{ const}$$

Suppose we consider equation (2a) and take derivatives:

$$\partial V_c = r_{21} \partial I_e + r_{22} \partial I_c.$$

Now keep V_c constant and we get

$$0 = r_{21} \partial I_e + r_{22} \partial I_c.$$

or

$$-\left[\frac{\partial I_c}{\partial I_e}\right]_{V_c} = \frac{r_{21}}{r_{22}} = \frac{a r_c + r_b}{r_c + r_b}. \quad (5)$$

Let us now define this quantity as

$$\alpha \equiv \text{short-circuit current gain.}$$

The "short-circuit" part of the definition arises from the requirement that V_c be constant (or zero).

$$\therefore \; \alpha \equiv \left[\frac{\partial I_c}{\partial I_e}\right]_{V_c \text{ const}}$$

It is now necessary to redesignate the previous α in the equivalent circuit by the symbol α_e for "alpha, equivalent circuit".

Then,
$$\alpha = \frac{\alpha_e r_c + r_b}{r_c + r_b} \approx \alpha_e.$$

or,
$$\alpha_e = \alpha - \frac{r_b}{r_c}(1 - \alpha).$$

The two quantities α and α_e are very nearly equal and are quite often used indiscriminately, but it should be kept in mind that they are different.

It is possible to convert the current generator in the equivalent-T to a voltage generator as shown in Fig. 3a.

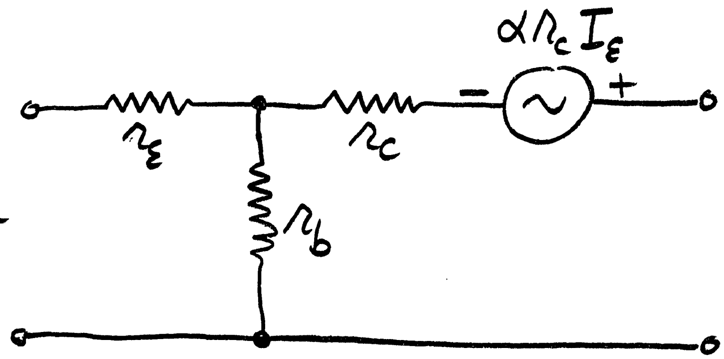


FIG. 3a

3.0 Hybrid Parameters

These are another set of transistor parameters which are used extensively in small signal work and increasingly of late in specifications. They are called hybrid because they make use of both current and voltage as independent and dependent variables. The equivalent circuit in terms of the hybrid parameters is shown in Fig. 4.

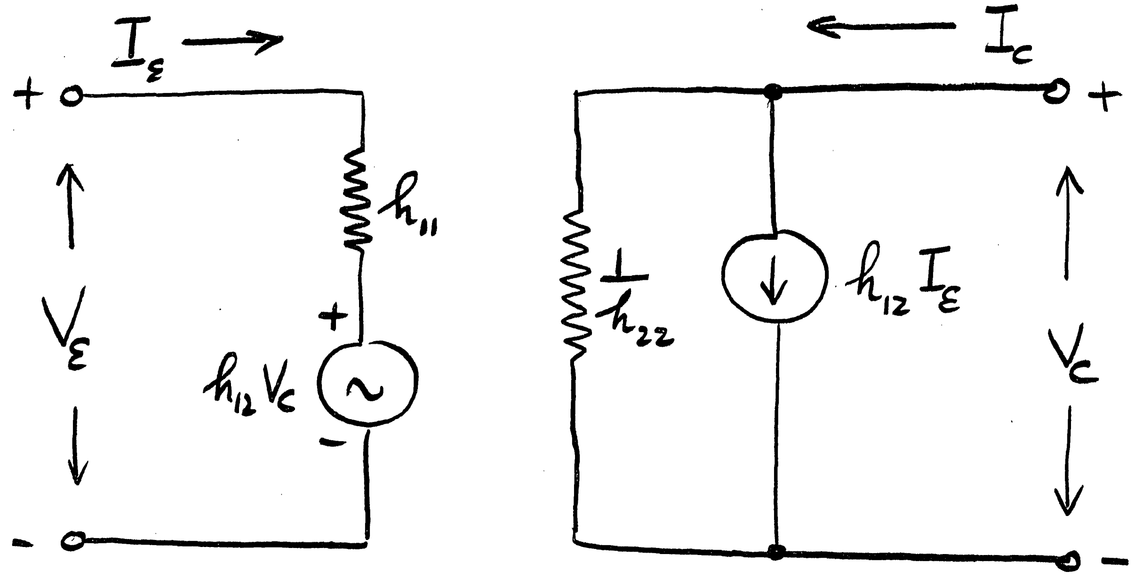


FIG. 4 - EQUIVALENT CIRCUIT USING HYBRID PARAMETERS

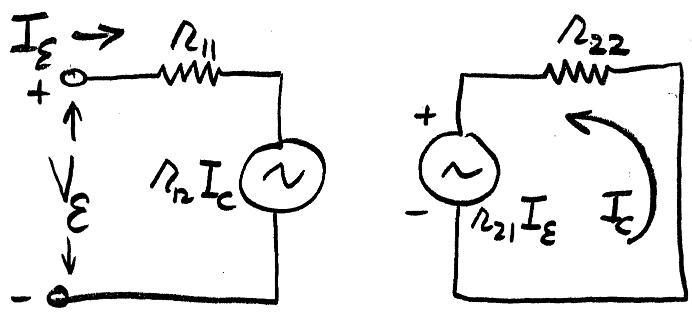
$$V_e = h_{11} I_e + h_{12} V_c \qquad (6)$$

$$I_c = h_{21} I_e + h_{22} V_c \qquad (7)$$

From equation (6) we can see by making $V_c = 0$ that,

h_{11} = <u>short-circuit input impedance</u>

Consider this in terms of open-circuit impedances as shown below:



From the collector circuit,

$$-I_c = \frac{r_{21}I_e}{r_{22}}$$

$\therefore$ the emitter generator $= r_{12}I_c = -\dfrac{r_{12}r_{21}I_e}{r_{22}}$

$$\therefore \quad V_e = r_{11}I_e - \frac{r_{12}r_{21}}{r_{22}} I_e$$

Thus the short-circuit input impedance,

$$h_{11} = r_{11} - \frac{r_{12}r_{21}}{r_{22}} .$$

or

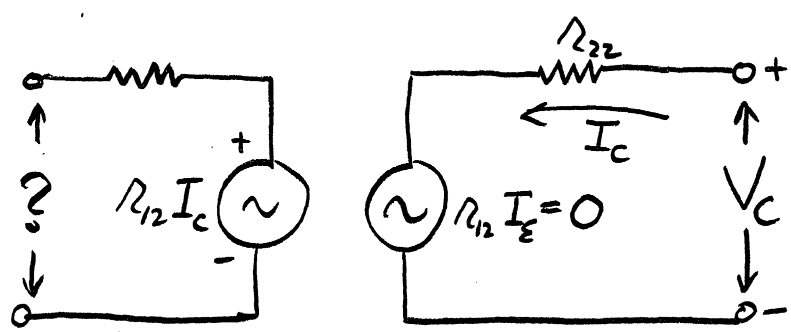
$$\boxed{h_{11} = r_e + r_b(1 - \alpha).} \qquad (8)$$

The short-circuit input impedance is also often expressed as $1/g_{11}$ where g_{11} is the short-circuit input conductance.

By making $I_e = 0$ in equation (6) we see that,

h_{21} = open-circuit feedback parameter.

In terms of open circuit impedances we can calculate this as follows:



From the collector circuit, $I_c = \dfrac{V_c}{r_{22}}$.

$\therefore$ the emitter generator, $r_{12}I_c = \dfrac{r_{12}\,V_c}{r_{22}}$.

$$\therefore V_e = \frac{r_{12}}{r_{22}}\,V_c$$

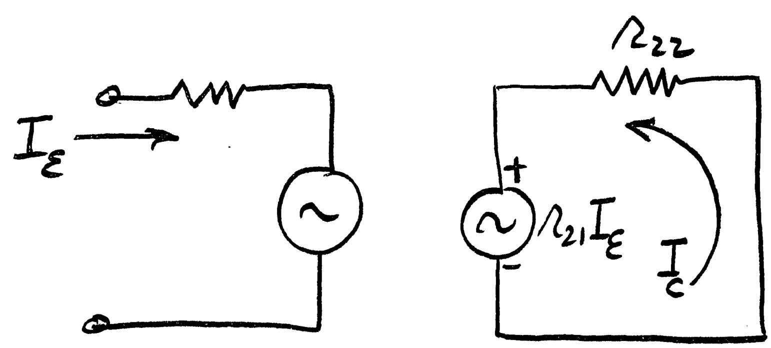
Thus, the open-circuit feedback parameter,

$$h_{12} = \frac{r_{12}}{r_{22}}$$

or

$$\boxed{h_{12} = \frac{r_b}{r_b + r_c}.} \qquad (9)$$

This quantity, the fraction of the collector voltage appearing at the open-circuited emitter, is also referred to as μ_{ec}. By setting $V_c = 0$ in equation (7) we find h_{21} = short-circuit transfer function. From the open-circuit impedance representation we get,



In the collector circuit $I_c = -\dfrac{r_{21}I_e}{r_{22}}$.

$\therefore$ the short-circuit transfer function,

$$h_{21} = -\frac{r_{21}}{r_{22}} = -\frac{\alpha_e r_c + r_b}{r_c + r_b} = -\alpha$$

$$\boxed{h_{21} = -\alpha.} \qquad (10)$$

By setting $I_e = 0$ in equation (7) we get h_{22} = open-circuit output admittance. This is just $1/r_{22}$.

$$\therefore \boxed{h_{22} = 1/r_{22} = \frac{1}{r_b + r_c}.} \qquad (11)$$

We can therefore redraw the circuit in Fig. 4 representing the hybrid parameters as follows:

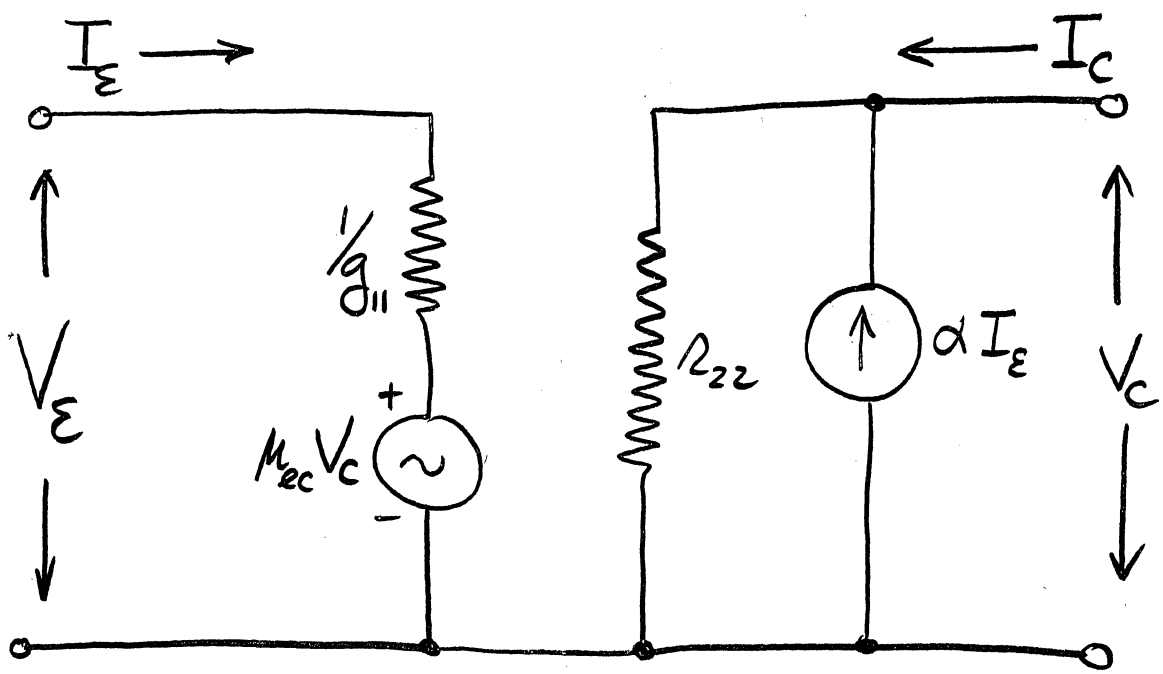


FIG. 5 - EQUIVALENT CIRCUIT FOR HYBRID PARAMETERS

$$h_{11} = 1/g_{11} \qquad h_{21} = -\alpha$$

$$h_{12} = \mu_{ec} \qquad h_{22} = 1/r_{22}$$

4.0 Early Modification of Equivalent Circuit

The measured values of r_c were found to be 1 or 2 MΩ, which is considerably below the theoretical value predicted for the equivalent-T circuit. J. M. Early of BTL resolved this difficulty by considering the change in collector space-charge width with collector voltage (now referred to as the "Early effect"). If we consider the potential diagram in Fig. 6 we see that as the collector voltage increases the effective base width decreases.

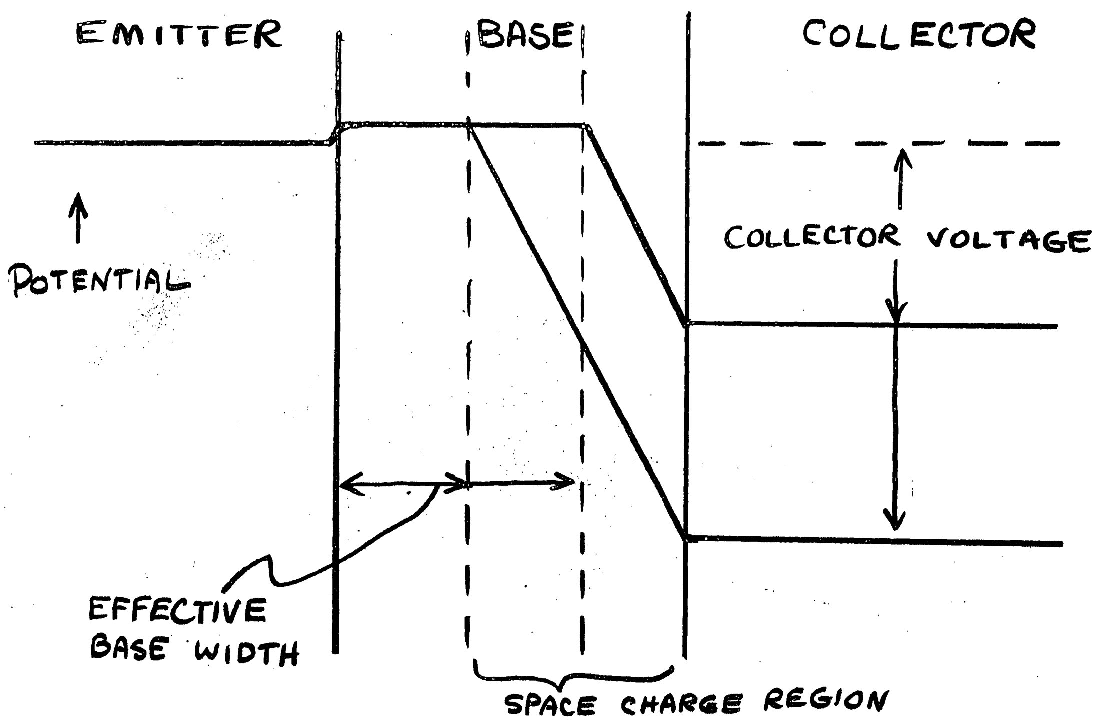


FIG. 6 - SPACE CHARGE WIDENING CAUSING BASE WIDTH TO DECREASE.

The effect of decreasing the base width is twofold: a decrease in hole-current loss by recombination of holes and electrons; and a decrease in the base impedance to hole injection by the emitter. Both of these factors tend to increase α, the current gain. Now, the open-circuit collector conductance,

$$g_c = \frac{1}{r_{22}} = \left[\frac{\partial I_c}{\partial V_c}\right]_{I_e} \approx I_e\left[\frac{\partial \alpha}{\partial V_c}\right]_{I_e} \approx 1 \text{ micromho.}$$

Thus, the low value of collector resistance is the direct result of space-charge widening or Early effect. The decrease in base width also causes an increase in base resistance but this is normally small.

The various effects mentioned above can be represented by the equivalent circuit below. The space-charge variation produces the collector resistance $1/g_c$ and the emitter voltage generator.

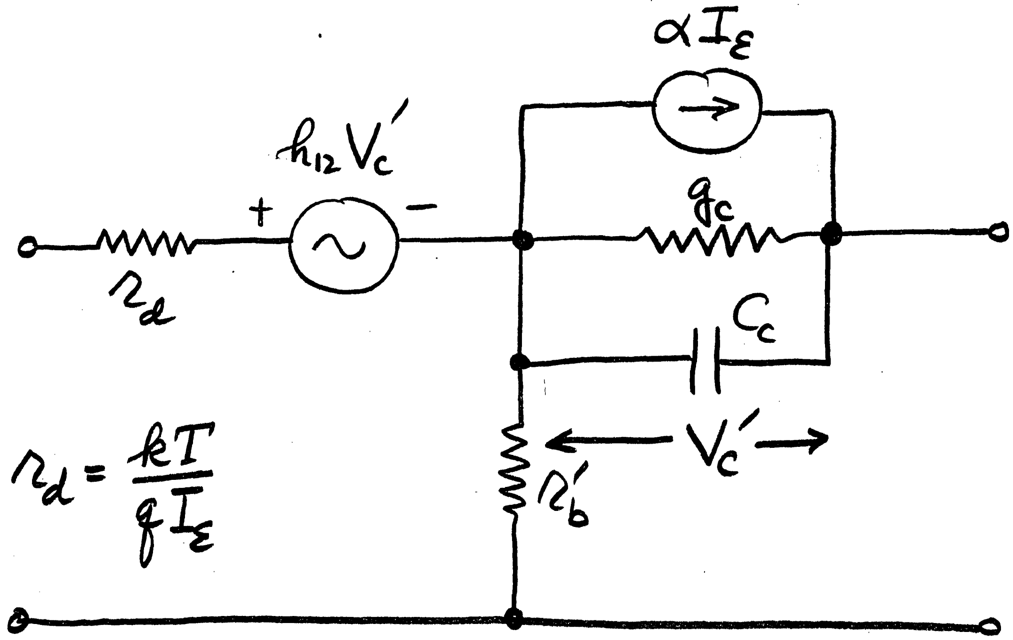


FIG. 7 - EARLY-MODIFIED EQUIVALENT CIRCUIT.

However, it is desirable to eliminate the voltage-dependent generator in the emitter. Doing this gives the equivalent circuit of Fig. 8.

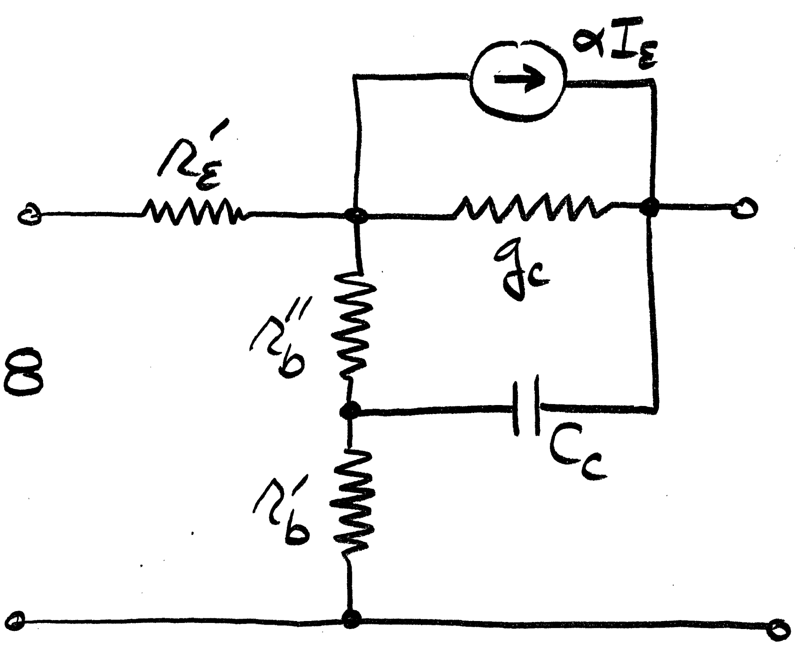


FIG. 8

The parameters in Fig. 8 are related to the previous parameters by the relations:

$$r_b'' = h_{12}\, r_c.$$

$$r_e' = r_d - (1 - \alpha)\, h_{12}\, r_c \approx \frac{kT}{2qI_e} = \frac{13\,\Omega}{I_e}.$$

$$r_b' = \text{spreading resistance.}$$

The feedback parameter $h_{12} = \frac{kT}{qw} \frac{\partial w}{\partial V_c} = \frac{25}{w} \frac{\partial w}{\partial V_c}$.

where w = effective base width.

Note that at low frequencies,

$$Z_b = r_b' + h_{12} r_c$$

while at high frequencies r_b'' is shunted by C_c and,

$$Z_b = r_b'.$$

Typical values for the parameters shown in Fig. 8 are the following for a pnp audio transistor:

$$r_b' = 300\,\Omega.$$

$$h_{12} = 4 \times 10^{-4}$$

$$r_c = 1\ M\Omega.$$

$$C_c = 40\ \mu\mu f$$

$$\alpha = .98$$

$$r_b'' = h_{12} r_c = 4 \times 10^{-4} \times 10^{6} = 400\,\Omega.$$

At any frequency where $\omega C > g_c$ the effective base resistance is r_b'. For the above transistor this frequency is

$$10 \times 2\pi\, fC = g_c$$

$$f = \frac{g_c}{20\pi C} = 40KC.$$

5.0 Equivalent Circuit for SBT

A final equivalent circuit worthy of note is a π-equivalent proposed by Philco as a characterization of the surface-barrier transistor. This is shown in Fig. 9: The parameter values are related to those previously given by the expressions below:

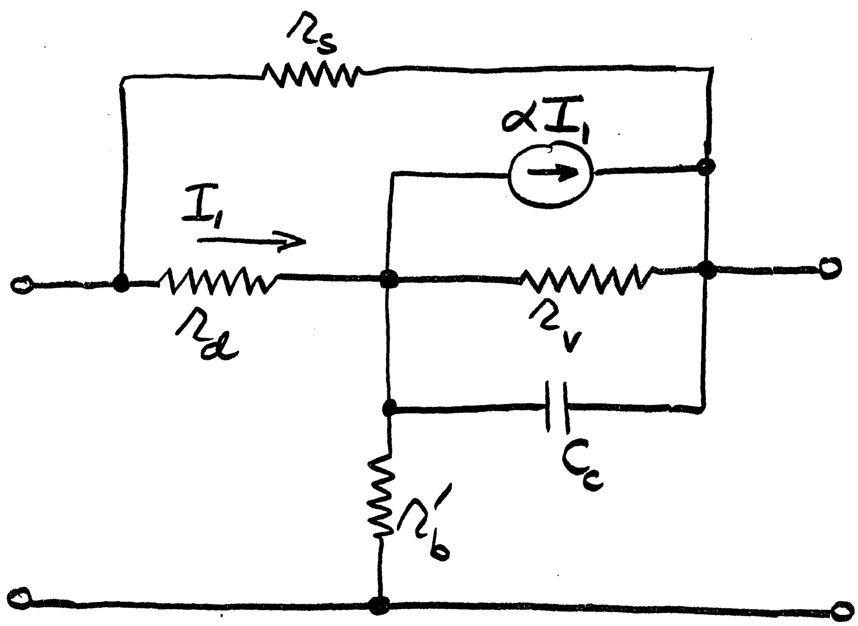


FIG. 9 - SBT EQUIVALENT CIRCUIT

$$r_d = \frac{kT}{qI_e} = r_e' + (1-\alpha)\, h_{12} r_c .$$

$$r_v = \left(\frac{r_d}{r_e'}\right) r_c .$$

$$r_s = \frac{r_d}{h_{12}} .$$

The various parameters required for these equivalent circuits can be obtained from sets of characteristic curves, which will be discussed in the next lecture.

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