OPERATIONS RESEARCH — THE SCIENTIST'S INVASION OF THE BUSINESS WORLD

BY DR. DEAN E. WOOLDRIDGE

THE ROLE OF MATHEMATICS IN OPERATIONS RESEARCH

BY DR. ANDREW VAZSONYI

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FOREWORD

by DR. PAUL STILLSON
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On April 28 and 29, 1955, the University of California, Los Angeles, sponsored the first Institute on Operations Research for Business and Industry on the West Coast. The purpose of this Institute was to introduce the philosophy, methods, and application of operations research to management level personnel in California.

As an applied science to aid in the solution of management problems, the industrial application of operations research methods is approximately four years old. Having its main impetus in the eastern and mid-western regions of the United States, this type of scientific activity has been somewhat limited in a geographical sense with very little participation on the west coast. Although the reasons for this apparent slow rate of diffusion across the country are not entirely clear, it was felt that one of the prime factors was the absence of an impartial information center for the distribution and dissemination of operations research and accomplishments. Therefore, in accordance with its established policy for providing educational assistance to industrial management, the University felt that an introductory conference on this subject would be both timely and informative. This undertaking was carried out by the College of Engineering through Engineering Extension.

Institutes on industrial operations research have been offered at several academic institutions in the past, starting with the initial exposition at Case Institute of Technology in November, 1951. At that time, the Operations Research Group at Case Institute invited the industrial leaders in their area to participate in a program which resulted in its now widespread application. Other institutions, such as Columbia University, Massachusetts Institute of Technology, and Johns Hopkins University, subsequently offered seminars, institutes and short courses to further stimulate the interest of industry. The western-most activity on operations research was held at Kansas City in April, 1954, by the Mid-West Research Institute. In all of these meetings there was a surprising absence of west coast representatives.

Therefore, this Institute was designed to serve in an indoctrination as well as an information capacity, and the program which was presented to the conference attendees was prepared for that level of management which could initiate operations research activities within their own companies. With this objective in mind, the more technical phases of methods and mathematical models were deliberately de-emphasized in favor of the philosophy of approach to management problems and the actual application of operations research in the form of case histories. In this regard, the Institute was in a more advantageous position than many of the preceding conferences, in that the accumulative efforts of the past four years could be reviewed, evaluated, and presented as proven accomplishments.

FOREWORD - Cont'd

Recognizing the diverse interests of a large group attending an introductory conference of this type, the program was further designed to allow for as much exploration into any of the phases of operations research as the individual attendee might desire. For this purpose, the two-day session was divided into morning presentations and afternoon discussions. Each speaker delivering a morning paper was assigned a discussion session, for a two or three hour period, in which related topics as well as the presentation topic could be further examined. In order to augment these invited speakers with a greater variety of experience, a co-discussant in the same field of application was invited to participate in each afternoon discussion. These afternoon discussion sessions were held as informal open forums.

We felt that the Institute was particularly fortunate in being able to draw from a distinguished group of operations researchers for both the invited morning papers and special discussants. Since most of the operations research activity has been in the east and mid-west, the majority of the morning speakers were from those areas. These speakers, as well as the discussants, represented as many different types of working background as are found in present operations research practice. Research activities in academic institutions, research foundations, military study groups, management consultants, and company research teams were ably represented by the group shown on the speakers list. In addition, the viewpoint of management toward operations research application was expressed by representatives of companies who are now in the process of conducting operations research programs and by those who have had the opportunity of evaluating previous efforts. Wherever possible, representatives of west coast institutions engaged in operations research activity were included in the program to facilitate post-conference advisement and discussion.

A special mention must be made of the Advisory Committee which was invited to assist in the preparation of the conference content. In order to determine which phases of operations research activity would be of greatest interest to management level personnel, the University staff invited top-ranking executives from leading business and industrial organizations in the Southern California area to review and comment upon all phases of the program. This twelve-man committee consisted of representatives from many major industrial groups, e.g., aircraft, petroleum, chemical, entertainment, etc., in order to insure adequate coverage of the diversified problem areas. Their criticisms and suggestions were instrumental in formulating the policy and direction of the Institute.

From the number of attendees and the favorable comments received at the conclusion of the sessions, we feel that the first Institute on Operations Research in Business and Industry on the West Coast was successful. We would like to express our appreciation to the outstanding group of speakers for their well-delivered presentations, to the Advisory Committee for their enthusiasm and cooperation, and to the conference attendees for their continued interest in scientific management.

OPERATIONS RESEARCH - THE SCIENTISTS' INVASION OF THE BUSINESS WORLD

by Dr. Dean E. Wooldridge
President
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In the dinner address, Dr. Wooldridge discusses the new relationship between the scientist and the business executive. Speaking from the viewpoint of the scientist, he describes the transition from the ivory-tower philosophy of research to what is now an accepted field for scientific endeavor. As an executive, Dr. Wooldridge recognizes the cautious attitude of the business manager toward this "invasion" and points to the potential benefits which could be derived from its acceptance.

I believe that almost any speaker who today addresses himself to the subject of operations research feels an almost irresistible urge to start by defining the subject. This is probably because, while he may know what he means by the term, he is not quite sure that the words mean the same thing to anyone else in the audience. Well, starting a speech with a definition of the subject isn't necessarily a bad idea, although for this particular subject there seems to be an unusual tendency for the defining process to get out of hand. There is more than one instance on record of a speaker whose definition of operations research continued until the chairman rang the warning bell at the end of the speech. I'm going to try to avoid this particular pitfall, but I'm afraid I too feel the need of starting with some kind of definition of operations research that is suitably tailored to the particular set of prejudices that I want to promulgate.

Rather than start with a formal and presumably uninteresting, suitably slanted definition, I propose to relate an episode of the last war that, to me at least, has always seemed to illustrate good operations research. Like so many good stories, I can't really swear that this one is true, but at least my attempts to run down its truthfulness have not resulted in convincing me that it is untrue.

The episode in question concerns a squadron of bombing planes that was stationed on one of the small islands in the Pacific in 1944. These planes were owned and operated by the Air Force, and the story revolves about some difficulties with the United States Navy. It seems that every once in a while one of the Air Force planes would be shot at, and sometimes hit, by naval vessels close to which the planes had occasion to fly. Now this might have been considered to be just a normal wartime occurrence, if it were not for the fact that the incidence of such unfortunate events was considerably higher for the bombing squadron in question than it was for other squadrons operating in the same general area. Finally, after some exchange of correspondence back and forth between the Pacific island and Washington, the problem was narrowed down to the low level of reliability of the IFF equipment in the aircraft.

Now to those of you who are not familiar with this term, IFF is an abbreviation meaning "Interrogate—Friend or Foe." The term refers to a black box in the aircraft that constituted in effect a special kind of radio receiver-transmitter combination. When any aircraft was sighted by a Navy ship, the ship would send out a special type of radio signal. If the aircraft in question was equipped with the allied IFF apparatus, then another specially coded signal would be returned to the interrogating ship, indicating that the aircraft in question was friendly; consequently guns would not be fired. But for some reason or other, the IFF equipment was not working very frequently for this particular squadron in the Pacific. Well, the routine efforts of maintenance on the Pacific island and the encouragement, and perhaps threats, sent out from Washington didn't improve the situation. So finally an operations research expert was sent to the Pacific island to perform careful detailed analyses, to localize the trouble, and, hopefully, to fix it.

Now, as you might imagine, any story that I would be telling at the beginning of an operations research talk would be intended to prove that operations research can indeed be very effective; so you will not be surprised to hear me say that within about two weeks of the arrival of the operations research expert on the Pacific island, the encounters between the aircraft of that base and Navy ships dropped in a rapid and spectacular fashion and furthermore the incidence of such encounters stayed practically at zero for week after week. Finally it was clear that the problem had been completely solved, and word was sent to the operations research expert to return to Washington. Upon his arrival, he was of course warmly greeted by his associates, who gathered around and congratulated him, and were most anxious to learn what unusual feats of analysis, higher mathematics, and electronics design he had been able to accomplish in such a short period of time to produce such a miraculous result. Since operations research men are always modest, objective scientists, the returning hero was not at all hesitant in explaining clearly what he had done. He said, "Well, after I had been there for a week asking questions and analyzing the subject, tracing electronic circuits, and making statistical calculations, I finally discovered what I thought was the problem. To cure it, I stationed a man at the end of the runway each time the aircraft took off, and as each airplane passed this man held up a big sign that we had painted. This sign carried on it the words, 'Turn on your IFF equipment.' From that time on we had no more difficulty."

Well, of course I have a point in telling a story like this. In the half hour we have ahead of us, I want to direct our attention away from some of the more sophisticated techniques that are occasionally employed by skilled operations research people to arrive at answers to some of the problems they encounter—the use of mathematical models, the Monte Carlo method, the theory of waiting, linear programming, and the like. Instead I want us to concentrate our attention upon what I at least consider to be the broad concept of operations research and what it is trying to do for its customer.

Now without getting into fine shades of meaning, it seems to me that the legitimate subject matter for operations research includes almost all aspects of any complex organization of men and/or equipment. In the example of my story, the operations research scientist was a trouble shooter. In some way a section of the complex organizations of the Pacific Air Force was not operating as effectively as was believed reasonable in terms of the over-all objectives of the mission. The basic assignment of the operations research scientist was to find out what was wrong and to make recommendations, whatever they might be, that might somehow improve the effectiveness of the operation. In the example in question, the highest mathematics required for proper solution of the problem was probably arithmetic, and very likely not much of that was needed, although it was likely that the man who was sent out to the Pacific island was fully equipped to deal competently with difficult statistical problems if such had turned out to be the principal points at issue. Conversely, an even less technical solution than the one told in the story might have been the result of the investigation. For example, the answer could have been that it was necessary to fire the Commander of the group. The point of all this is that the subject matter of operations research is very broad—it is the effectiveness of the complex organization of men and machinery in achieving its over-all objectives, whatever they might be.

But if you accept the rough but very broad definition of operations research that I have just developed, then you have a right to ask, "What is the difference between the task of the operations research expert and that of the manager of the enterprise? Isn't the manager generally earning his salary on the basis of conducting the operation of the enterprise in such a way as to make it most effective in terms of the over-all company objectives, and isn't it his job to run down inefficiencies and modify operations and procedures as required in order to maximize effectiveness? What is the operations researcher up to when he comes to the manager of a company and asks for an assignment? Is he simply saying to the manager, in effect, 'anything you can do, I can do better.'?"

Well, these are questions to which those of us who are engaged in the selling of operations research have a very high sensitivity. When such questions are asked, we frequently fall all over ourselves to try to reassure the manager that the last thing in the world that we want to do is to give him the impression that we are criticizing him of inefficient management, or to cause him to feel that we are trying to step in and take over from him the job of managing his company. We then generally go into quite an exposition to show what the differences are between operations research and management in an attempt to prove that what we can do is to supplement the manager, but under no circumstances are we really setting out to do the same thing. Well, I think we frequently defeat our own purposes by this kind of reaction. I believe that a more accurate and certainly a much simpler reply to the manager who asks if the operations research people are not representing that they can step in and do a better job than the manager is doing of some of the functions he is being paid for is just a plain, unembellished "Yes." Of course, that is what we are saying. Oh, I'll admit that there are some management decision areas that are not very well suited to an operations research approach. It is not likely, for example, that the objective, presumed scientific analysis performed by an operations research team could ever develop convincing arguments to prove to the management of the Salvation Army that they should give up their charitable objectives and go instead into the manufacture of poison gas. In other words, there are certain rules of the game that constitute the reason for existence of the company and its basic aims and ambitions that are not properly the subject matter for objective analysis. Admitting that type of exception, however, I believe it is almost true to say that everything else the manager does in operating a complex organization of people and equipment and every other kind of decision he makes are basically suitable subjects for operations research.

Therefore, those of us who are trying to sell operations research, I believe, should face up to the fact that when we go to a manager with our sales pitch we are in effect saying to him, "Look, we don't believe that you are managing your company as effectively as it could be managed in terms of your over-all company objectives. Put us on your payroll for a while and we believe that we can affect an improvement."

Well, I don't know how many operations research people will go along with the rather blunt interpretation I have just made of what it is that operations researchers are trying to sell business people, but I haven't any question at all as to how most managers feel after listening to our propaganda. By and large, if they don't end up being hopelessly confused about what operations research is all about, they generally arrive at the conclusion that what the operations researcher is trying to do is to tell the manager that he isn't doing the best possible job of management and that operations research can improve his performance. This poses the manager with a very interesting problem. The only thing he can be absolutely sure of, if he authorizes operations research activities either by groups within his company or by outside consultants, is that he will end by spending money for their salaries. No one, least of all the legitimate operations research scientist, can or will guarantee in advance that the results will be worth the expenditure.

There are two ways the manager can react to this kind of proposition -- I might more accurately say there are two ways that managers do react. One type of reaction is simple, straight-forward, and to the point. In this type of reaction, the manager says either to himself or outloud, with or without profanity, something about as follows. "I'm having a hard enough time keeping this company in the black as it is. The last thing in the world I need is to pay some long-haired scientists to come down out of their ivory towers and tell me how to run my business." Well, this point of view no doubt has some merit. It nicely eliminates one decision that the manager would otherwise have to make. It prevents a certain amount of confusion in the organization that always results when operations research teams go to work, and it certainly saves the money that would have to be paid for the salaries of the operations research group. It has only one important drawback. Well, two perhaps. One is that if the operations research people were right, the company is not going to get the benefit of improved operations. The other drawback is that if I were to accept this point of view, I would have no excuse to go ahead with the rest of my remarks, and I would have to sit down before my time is up.

Fortunately, from my point of view at least, a great many managers react in an entirely different way to the suggestion that an operations research team can improve the management of their enterprises. Each of this class of managers goes through something like the following process of reasoning. He starts with the observation that he is sitting on top of quite a complex organization involving hundreds or thousands of people. These people are divided into various groups, each group living in accordance with a certain set of rules, regulations, and procedures characteristic of the group and related to the other groups by another set of rules, regulations, and procedures. There are accountants, production control people, manufacturing people, engineers, purchasing people, budgeting groups, and so on. Usually the manager of the enterprise is thoroughly familiar with what goes on in only one or two of these major sub-empires, and he is so busy that he has difficulty in maintaining himself current even with respect to these one or two areas. In all other major areas of activity, the manager knows that he himself does not well understand everything that goes on. From time to time major problems have arisen in the operation of the company that appeared to be localized in one or another of the functional areas. On some of these occasions, the manager has attempted to probe into the activities of one or more of these groups and learn enough about their detailed operations to permit him to form his own personal conclusions as to what his problems were. Usually he has been frustrated by such efforts, primarily because he has encountered such ramifications, complexities, and multiple sets of human and functional interrelationships that he simply could not accomplish his objective of self-education in the limited time he could spend on the matter. But this kind of manager has the recollection of such events in his mind and he therefore suspects strongly that there are areas in his company where the methods and procedures governing the day-to-day operational and decision-making activities are not well tailored to the special requirements of his company's business.

At this point let me assure you that the kind of manager I am talking about is by no means rare. I believe he is typical of the managers of nearly all large and many medium-sized enterprises. Such managers feel frustrated about some aspects of their company operations that they are pretty sure are not as efficient as they should be, but they simply don't know where to turn to get an impartial, objective appraisal that takes account of broad company aims and properly subordinates the local interest, prejudices, and established traditions that may currently govern some of the company activities.

Such a manager does not feel insulted when it is suggested that some of the operations of his company may be less efficient than they should be. Furthermore, since he is still convinced that if he personally could only spare the time to dig into the operations he would be able to turn up with major improvements, it is not hard for him to accept the basic idea that an operations research team might be able to accomplish much the same result. Then too, he knows that it isn't necessary that the operations research team be composed of the world's greatest geniuses, for he is convinced from his own experience that the application of common sense and reason combined with a proper appreciation of basic company objectives can go a long way in generating efficient operational and decision-making methods and procedures. To sell this type of manager on an operations research program, it is mainly necessary to convince him that the team that can be put on the problem will be composed of competent men of reasonable intelligence who can bring to bear an objective, quantitative approach, and that the team furthermore will do its work in a sufficiently smooth and tactful manner as not to produce too much disruption in the day-to-day affairs of the company. If these conditions can be met and if the price isn't too high, then this type of manager is going to buy operations research.

Well now of course you will have observed that while I have been making use of the device of talking about what one class of managers will do when confronted by the operations research pitch, what I have really done is give you the operations research pitch myself by making it appear to be obviously a good thing. My thesis, as you can see, has been that almost any complex business involving lots of people is likely to include procedures, methods and ways of making decisions that are not tailored to the actual over-all company objectives as well as they might be. I have made much of the idea that any team of reasonably intelligent and experienced men, with an objective and quantitative approach to problems can, if given the time and the opportunity, dig into a situation of this sort and turn up with recommendations for improvements that probably will more than pay for the salaries of the operations research team, and occasionally may even result in quite a major increase in the effectiveness of the company.

In talking about operations research in this way, I have hoped to clear up a bit of the confusion that sometimes seems to surround this subject. At the same time, however, I may have succeeded in generating a little more uncertainty as to the difference between operations research and old-fashioned management consulting. After all, haven't the management consultant firms for years engaged in doing exactly the kind of thing that I have described -- going into business and industrial establishments, analyzing their operations, and making recommendations for changes to improve over all effectiveness? Here again, I'm afraid I'm going to be a bit unorthodox from the point of view of the operations research fraternity by asserting that, in my opinion, management consulting and operations research are terms that have nearly the same meaning. Having said that, however, I will hasten to add that there are some important differences in the work that has been done in the past under the label of management consulting and the work that is going on today more generally under the label of operations research. The essence of the difference has to do with the professional background of the people who are doing management consulting under the name of operations research. The new groups consist largely of people who have trained as scientists. Probably the most important single fact you must know to understand why you are beginning to hear so much about operations research these days and why it is becoming such a powerful new tool of management is that within recent years scientists have decided that the operations of complex aggregations of men and machines, as typified by business and industrial establishments, constitute legitimate subject matter upon which self-respecting scientists can spend their time.

Those of you who do not yourselves have a technical background may find it difficult to appreciate the significance of the remark I have just made. Since this point is really quite vital to an understanding of what is going on in operations research today, I want to dwell on it for a few minutes. The subject matter we must deal with in trying to understand this situation is essentially a certain kind of snobbishness that characterizes scientists. Now if this sounds like harsh criticism, at least I can say that it is self-criticism as much as anything else, for I started my professional career as a Ph.D. in Physics about twenty years ago, and I believe my attitudes were quite typical. When I came out of graduate school clutching my bright and shining Ph.D. degree, I had had ingrained into my thinking the certainty that there were only a few limited fields that were worthy areas of occupation for a Ph.D. in Physics. As a matter of fact, there was some serious question in those days as to whether there was any field that it was really respectable for a physicist to work in except nuclear physics. In those days nuclear physics did not mean atom bombs, of course. It had to do with learning more about the fascinating and rather mysterious laws of nature that governed the construction and behaviour of the nuclei of atoms. I remember I had to make a difficult, and almost degrading personal decision to accept a position with Bell Telephone Laboratories, where, instead of working on nuclear physics, I found my self doing research on various solid state matters -- how electrons behave inside of crystals, and the like. This field of research involved quantum mechanical considerations and in other ways was adequately respectable so that I felt no danger of being shunned by my professional contemporaries for accepting such an assignment. However, it was about as far as a self-respecting physicist could go in those days.

Then came the war. The war had a profound psychological effect upon scientists such as myself, for many of us were plunged into problems of a type that we would never in peacetime have considered to be suitable subjects for us to work on. The results were most interesting. To begin with, because we had a good training in basic fundamentals, we turned out to be fairly effective in such matters as the development of radar, electronic computers, and the like, where the necessity of devising entirely new techniques frequently put technical success completely out of the reach of conventional engineering types of approach. The aspect of the situation that was less predictable and therefore more surprising, however, was that by and large the scientists discovered that this work was interesting and challenging. Pretty soon we learned that the job of inventing radar and computing equipment contained problems that were every bit as difficult and as fascinating to solve as the problems associated with atomic nuclei or solid state physics. This, let me emphasize, was quite a revelation to many of us. As the war went on, this broadening process continued. Pretty soon, we found ourselves involved not only in the design of new equipment, but also in the problems associated with the use of that equipment in military establishments. We had to concern ourselves with the necessity of providing output data from the equipment that human beings could understand and deal with, perhaps as they were flying fast aircraft. We had to concern ourselves with the problems of maintenance and reliability and with complicated practical problems of logistics and supply. We found ourselves also involved in sales activities, whereby we had to present to basically nontechnical people what it was we were trying to do and why we should be permitted to do it. And so it was that literally thousands of men trained as scientists or research engineers, who normally would never have considered working in other than a few highly technical areas, were forced by the exigencies of war to see some of the outside world and to learn that they could find interesting and challenging problems in broad operational situations in which the purely technical content might be fairly small. Now the individuals who went through that experience have, as a consequence, been available for a broad range of assignments since the war. But something perhaps even more important than that has occurred. Because of the influence that these men have had on the curricula and on the students of the universities where some of them have gone back to work and to teach, and the influence they have exerted through professional societies and the many other ways by means of which scientists keep in touch with one another, the general attitude toward life with which new scientists start out into the world is now quite different than it was before the war. Let me assure you that the effect of this whole process has been quite revolutionary. One of the important practical results is that since the war major weapons systems development projects have come along rapidly because good scientifically trained people have been willing to work on these programs. The consequence of greatest significance for the discussion this evening, however, is that today a man trained as a scientist can still hold up his head professionally and not be looked down upon by his contemporaries if he chooses to go into operations research and concentrate his scientific training on the analysis and solution of problems that arise in large human organizations. Such a situation, unthinkable a few years ago, is a direct consequence of the processes set in motion during World War II.

What is new about management consulting or operations research, then, is that scientists have begun to get into it. It just happens that scientists like the name operations research better than they do management consulting. (We scientists have not entirely lost our snobbishness even yet.)

Well, I hope you have found this history of the metamorphosis of scientists of some interest, and I further hope you may feel that it helps to explain some of the aspects of this whole management consulting -- operations research business that appear confusing to many people Apart from that, however, of what significance to the management of companies is this entry of the scientist into his domain? If there something here that really presages a higher caliber of management consulting than has been available in the past? Or is all of this talk about operations research simply a consequence of the scientist trying to pat himself on the back? Well, maybe there is some of the latter in the picture, but I believe that there is also good solid reason for management to be happy about the increasing availability of scientists for operations research assignments. Since I have confessed that my training is as a scientist, you will probably make your own allowances from this point on as I tell you why it is that I believe scientists are in a favorable position to contribute in this field, but, nevertheless, I must proceed to give you the benefit of my prejudices.

I believe that the scientifically trained man brings to business management several very important qualities. The first of these qualities is professional objectivity. One of the traditions that has been pounded into the head of everyone who emerges as a graduate scientist from a legitimate institution is that he must try to be objective in his approach. He must avoid depending upon authoritative opinions of others as his reasons for conclusions. He must scrutinize the raw data on which conclusions have been based. He must painstakingly check the logic at every state of the game before he arrives at his own conclusion.

Now there are two admissions that must be made to a group such as this in connection with the objectivity of scientists. The first is that of course scientists have no monopoly on objectivity. Most good managers are pretty objective also. The second admission is that scientists don't always apply this professional objectivity to their politics or their private lives. However, this is by no means the only instance of a class of men who operate their business in accordance with very rigid professional rules and standards, but who do not always carry over such professional rules to their personal lives. I'm afraid scientists are no different than other people in these respects. The important thing is that in their professional lives they are heavily indoctrinated with the idea of objectivity. The standing that they have among their associates depends in a major way upon whether their work gives evidence of an unprejudiced, carefully substantiated objective approach. Most other classes of people are not subject to the same kinds of professional compulsions to be objective as are the scientists. As a consequence, on the average, the scientifically trained investigator of a business or industrial operation does a better job than others in separating fact from opinion and in arriving at conclusions that are properly related to the actual state of affairs under study.

A second attribute that is common to scientifically trained people is that of quantitativeness. The scientist is trained to work with magnitudes and not just with qualitative effects. The scientist typically is confronted with situations in which a variety of physical phenomena occur simultaneously. He learns at an early date that the proper approach to any new problem is first to determine which factors are the major ones and which are the minor ones, and then to devote his principal attention to the important factors. A qualified scientist does not seize upon a narrow aspect of a problem that accounts for 10% of the phenomena he is investigating and concentrate his attention on that. He looks for the aspect of the situation that accounts for 90% of the phenomena and concentrates his attention upon that instead. Such a quantitative approach is also not unique to scientists, but they have the advantage of having had this sort of thing drummed into them in their formal training and in their professional experiences. For years and years, they have spent most of their time in arriving at quantitative, numerical, solutions to complex problems. Such a quantitative approach is of great importance in the analysis and resolution of operational and procedural problems that arise in a complex human society. The one thing that can usually be depended upon is that the problem that really is responsible for inefficient operations will be camouflaged by a large number of inter-related matters that can be differentiated from the major issue only by a quantitative evaluation of their relative effects on the attainment of the over-all objectives of the operation under study.

Another pertinent quality of the scientist is his capacity for studying and learning new fields. In his day-to-day activities, the scientist has frequent need to acquire an understanding of the important elements of some new subject that he may never before have encountered. This capacity is the principal tool by means of which operations research scientists, whose past training may not have included business and industrial methods and procedures, have been able to demonstrate an ability to equip themselves with a good understanding of these nontechnical matters in what sometimes seem to nonscientists to be a remarkably short time.

There is one other advantage many scientists have that, when added to these other points, gives them an unusually favorable position for certain kinds of management consulting activities. As everyone today knows, we are in the early stages of what will within the next twenty years begin to look like a second industrial revolution, characterized by the wide-scale application to business and industry of automation techniques, both for the operations of factories and for the handling of data processing assignments. The equipment that is being developed and applied to these tasks is, for the most part, of a highly complex character, and it is primarily electronic in nature. It just so happens that the broadening experience that so many scientists went through during the last war was accomplished in connection with the development of precisely these electronic techniques that are now beginning to be applied to business and industry. This was especially true of physical scientists and electronic research engineers, who today comprise one of the most important new classes of professional entrants into the field of operations research. But it is in the application of some of these newer tools of automation and data-processing that an important part of the growing need for operations research arises. While the manager of a company may or may not decide in favor of an operations research program in the normal course of his business, he has practically no alternative on the occasion of introducing the newer techniques of automation. The very nature of these new powerful tools frequently makes operational procedures simple that were completely impractical before, and at the same time practically requires the elimination of some procedures that in the past were reasonably efficient.

And so, not only does the objective, quantitative approach that results from his professional training put the scientist in an unusually favorable position to be effective in operations research, but also the growing impact of automation techniques in business and industry places an unusual premium on the physical scientist who can deal effectively with the newer electronics techniques as well as with generalized operation problems.

The time has come for me to bring this talk to a conclusion by summarising the points I have been trying to make. As you can see, I have been painting with a broad brush this evening. By encouraging you to think of operations research as simply a quantitative, objective approach to the solution of almost any complex management problem, and in minimizing the differences in meanings between the terms management consulting and operations research, I have consciously and deliberately over-simplified the situation. On the other hand, my generalizations have been much more nearly right than wrong, and by employing them I have hoped that we might succeed in clearing up some of the confusion that seems to surround this much discussed and little understood subject of operations research.

The principal points I have tried to make, in addition to the approximate synonymity of the term operations research with quantitative objective analysis of management problems, have been two in number. The first was that the thing that is new in this field of management consulting or operations research is that scientists have decided that such activities are respectable and, as a consequence, are getting into the field in large numbers. My second point was that this is a good thing. I have tried to convince you that, as a class, scientists do a better job than most people of separating the major and minor factors in a complex operational situation and applying to them an objective quantitative analysis to arrive at recommendations for company procedures or decision making methods that best serve the major over-all objectives of the organization. In the special class of situations which may be expected from now on to be the source of a steadily increasing fraction of operations research requirements, arising out of the application of automation techniques to business and industry, I have pointed out that operations research groups containing physical scientists or electronic research engineers should be uniquely productive.

The scientist has now decided that the business man is a respectable partner. There are some indications that the business man is willing to be wooed and won. If a marriage does indeed result, there is every likelihood that the union will be a happy and fertile one.

THE ROLE OF MATHEMATICS IN OPERATIONS RESEARCH

by Dr. Andrew Vazsonyi
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The mathematical reasoning so essential in Operations Research is accessible to all management personnel with or without mathematical training. Dr. Vazsonyi presents a simplified mathematical model useful in production and inventory control utilizing matrix algebra. This presentation was explicitly developed for and repeatedly tested on business personnel with limited scientific background.

INTRODUCTION

What is operations research? This question has been with us ever since we have been talking about operations research. Each of us have a more or less articulated answer. You just heard an exposition by Mr. Hurni which should give you a good idea of what operations research is. However, we are not really in a position to tell you exactly what operations research is, as each of us have a somewhat different definition. Some people feel that operations research is something quite new, and has never been done before; some other people feel that there is really nothing much new about it, it is just an extension of current industrial practices. Some people classify industrial work that they consider very good as operations research, and work that does not appeal to them as not being operations research. A lot of time and effort was expended in answering this question, but the result is, generally speaking, frustrating. However, we should not be disturbed by this state of affairs; something new is being developed and at this stage of the game it is to our benefit to have a variety of opinions.

There is, however, one aspect of operations research on which there is fairly unanimous agreement. This aspect is the use of mathematics in operations research. Suppose you are thumbing through a stack of magazines dealing with management problems. Operations research aims at the solution of management problems, and, therefore, let us assume that the Journal of Operations Research Society of America is one of the magazines you are thumbing through. You will find a striking difference between the appearance of the Journal of Operations Research and the other magazines; operations research papers (perhaps not all of them) are full of mathematical equations. The other magazines on business, on the other hand, contain verbal descriptions. There are some people, of course, who say that there is too much mathematics in operations research, and that this is a weak point of operations research; this might be so. However, it is not premature to say that mathematics is here to stay in operations research. Perhaps as we go on, the mathematics will become more simple and will be more understandable to the businessman; however, a good fraction of the papers on operations research will contain mathematics in the future, too.

A legitimate question on the part of the businessman is whether the use of mathematics is really necessary, or is mathematics used in business because scientists are used to employing mathematics in other fields? The purpose of my talk is to prove to you that the use of mathematics in business is not a fad, but it is a sign of coming things. I do not claim that all operations research contains mathematics; I do not claim that all business situations should be handled with the aid of mathematics. However, I do claim that there are many business situations where mathematics makes it possible to describe and comprehend the fact of the business situation better than any verbal description can hope for. The fact of the matter is that mathematics is a language capable of describing certain business situations.

There is perhaps no other scientific subject of which there is so much confusion than on mathematics. Most of us have been exposed to certain aspects of mathematics in school, and have developed a healthy dislike and dread for the subject. Very few of us are proud of not knowing some particular subject, but mathematics is an exception. Again and again I meet people who keep on emphasizing that they don't know any mathematics, that they will never be able to understand any mathematics, and that it is quite hopeless to try to teach them any mathematics. They seem to be quite proud of this state of affairs, a thing I find somewhat puzzling. This attitude is perhaps based on the deep belief that mathematics cannot be of any use in business.

On the other hand, there are some people who take a different attitude: they claim that business is full of numbers. business deals with profits measured in dollars, with costs again measured in dollars, etc. Therefore, mathematics should be quite useful in the business world. This is an argument that most mathematicians would face with a shudder.

The only way to understand the use of mathematics is to use it. This leads, however, to a vicious circle. What comes first, the chicken or the egg? Let us recall at this moment that certain mathematics has been beaten into all of us. Most of us know how to multiply and divide. You might say that this is not much of mathematics. Our ancestors in the Sixteenth Century had a different point of view, as there were some universities that declared that all students would learn how to multiply, but the better ones will learn how to divide. If you go 2,000 years back, to the Roman World, you recognize that very few people know how to multiply or divide. This should not surprise us, because if we tried to carry out a division with Roman Numerals, we would recognize that a genius is required to perform such an operation. Why do we know how to multiply and divide? I don't think any of us particularly enjoy learning these operations in school; but, we had to learn them because if we did not we would have flunked out in school.

It is entirely possible that the use of mathematics in business will take a similar course. As time goes on we will get more and more used to the idea that mathematics is useful. The things that look very difficult to us today will become second nature to the manager of a plant 50 years from today.

What is the reason that scientists have come to use mathematics? Let us spend a few moments in examining a problem of physics. Suppose we want to know how far a falling stone is going to go in a certain length of time. People who observed this phenomenon tell us that in one second a stone falls 16.1 feet, in two seconds 64.4 feet, in three seconds 144.9 feet, etc. Let us put this into table form:

1 second		16.1 feet
2 seconds	4 x 16.1 =	64.4 feet
3 seconds	9 x 16.1 =	144.9 feet
	.	
	.	
	.	
½ second	¼ x 16.1 =	4.0 feet
	.	
	.	
	.	

On the basis of this information we can compute how far a stone will fall in any length of time. The rule implicit in the table could be formulated in words, but let us not bother to develop this statement.

The situation of the falling stone can also be described with the aid of an equation. This equation can be written as

$$D = 16.1\, t^2 \tag{1}$$

where D is the distance, in feet, that the stone falls, and t describes the number of seconds. You will readily see that this description of the problem is more precise and concise than describing the phenomenon with the aid of the table.

However, this is not all. Suppose we want to know how long it takes a stone to fall a hundred feet. Presumably you could work out the answer to this problem without mathematics. However, it is much simpler to write the formula

$$t = \sqrt{.06D} \tag{2}$$

which tells us the time it takes for a stone to fall a distance of D. If we want to get the answer for 100 feet we have to substitute 100 where the letter D appears, take a square root and then we get the number that it takes 2.4 seconds for a stone to fall a hundred feet.

You see here that mathematics can help quite a bit in such a problem. The real advantage comes, though, when we deal with more complicated phenomena. In order to handle the problem with a verbal description, we need to construct a pyramid of arguments, which becomes more and more complicated and confusing as the phenomenon becomes more complicated. Pretty soon we get to the point that the verbal argument loses its usefulness, and all we have is a confused state of affairs. Mathematics makes it possible to economize in thought, makes it possible to deal with the problem in question effectively.

You are perhaps beginning to be bored with this description of the falling stone. It is quite far-fetched to claim that there is a relationship between a falling stone and a business problem. We all know that business cannot be described with the aid of formulae; there are too many intangibles involved, there are different personalities, organizational problems, motives involved. How can anyone claim that all these things can be put into a formula?

I claim nothing of the sort. Even in the case of the falling stone there are a great many things that I left out of the equation. What about air resistence? This will depend a great deal upon the particular shape of the stone, and the particular surface roughness. All these things are completely ignored in the formula; however, it is possible to develop a more elaborate formula for the motion of this stone. This formula would take into account air resistence, but probably even this would not take into account all the various ramifications of the problem.

A very similar situation exists when describing business problems with mathematical equations. Equations will always describe only certain aspects of the problem, and a great many things will be left out. The point, however, is this: the objections raised against using mathematics in business are quite similar to the objections that can be raised against using mathematics in physics, and, in fact, these same objections were raised hundreds of years ago in connection with physical problems.

Let us now stop talking about problems in physics and let us turn out attention to business problems. Imagine a hypothetical factory producing some rather complex assemblies, similar to airplanes or radar sets. There might be thousands of these articles produced on assembly lines and in machine shops. Imagine that master schedules for the shippable items are set well in advance, but are subject to periodic changes. We propose to illustrate the use of mathematics in business by showing how mathematical methods can help in solving some of the problems this factory might face. Certainly the manager of production control in this factory wants to know how many parts and assemblies he should manufacture to meet this particular shipping schedule. Also, he wants to know at what time he should manufacture these various parts. We propose to show that this problem can be described and solved with the aid of mathematical equations. We also propose to show that with the aid of mathematical equations it is possible to determine the machine and labor hours imposed on this factory by this particular master shipping schedule.

Systems of equations, or formulae, that describe a business problem are customarily referred to as the mathematical model of the business in question. The problem of how many parts are required to meet

ASSEMBLY DESCRIPTION: PANEL — MAKES ASSEMBLY: 435090012 — PAGE

N.A. QTY.	PART NUMBER		DESCRIPTION
3	420990309		BUSHING
1	435090012	1	PANEL BLANK
2	435090012	2	ANGLE
1	435090012	7	ANGLE
5	99967C098		RECEPTACLE
10	AN426AD3		RIVET
8	AN426AD4		RIVET

This sheet refers to the assembly "panel" which is assigned the part number 435090012. This panel is made up of seven different articles which could be subassemblies or parts. The panel requires three bushings 420990309, and one panel blank 435090012-1, etc. Each assembly will have a similar sheet.

CMC — LAST PAGE

Fig. 1. Assembly Parts List.

a certain shipping schedule is often referred to as the problem of parts listing. Therefore, we proceed now to establish a mathematical model; that is, a system of equations, that describes the problem of parts listing in a manufacturing firm.

A MATHEMATICAL MODEL FOR PARTS LISTING

The basic information required for production control is usually listed on so-called assembly parts lists. An example, shown on Figure 1, refers to a Panel with part number 435090012. This "Makes Assembly" is made up of seven different articles as shown on the parts list under the heading N. A. QTY. (next assemblies quantity). It is shown how many of these articles are needed. Thus, the bushing part number 420990309 is required in a quantity of three for each of the panels 435090012.

There would be a similar sheet for every assembly, and in our hypothetical factory there would be perhaps a few thousand of these sheets. Our first problem, then, is to put this information into concise mathematical form. Before we do this it will be useful to think of this problem in terms of a graphical representation. Figure 2 shows a simplified situation with three top assemblies and a total of nine articles.

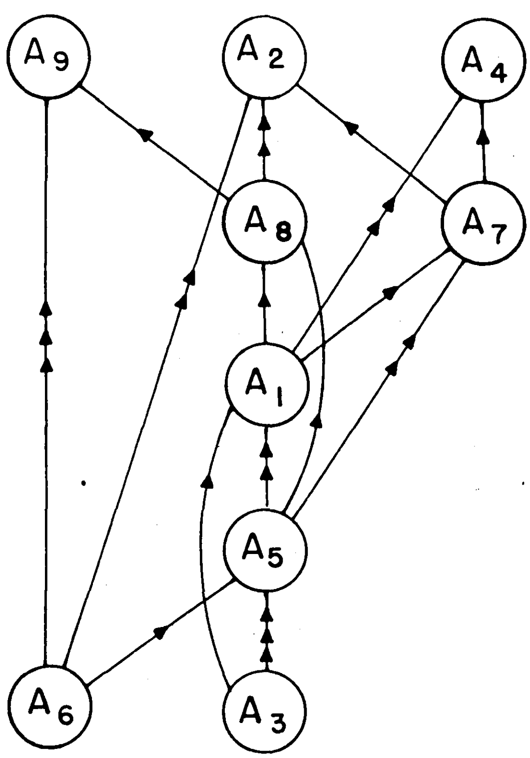

Fig. 2. The Gozinto Graph is a pictorial representation of the parts requirements. The next assembly quantities can be observed directly by counting the arrows on each connecting line. Total requirements cannot be observed directly but can be deduced.

The Next Assembly Quantities are shown by the arrows on the figure. The various Next Assembly Quantities shown on Figure 2 are represented in a tabular form in Figure 3, and this Next Assembly Quantity Matrix is a concise mathematical representation of the information contained in the assembly parts list.*

	1	2	3	4	5	6	7	8	9
1	0	0	0	2	0	0	1	1	0
2	0	0	0	0	0	0	0	0	0
3	1	0	0	0	3	0	0	0	0
4	0	0	0	0	0	0	0	0	0
5	2	0	0	0	0	0	2	1	0
6	0	2	0	0	1	0	0	0	3
7	0	1	0	1	0	0	0	0	0
8	0	2	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0

Fig. 3. Next Assembly Quantity Table. This is a concise mathematical representation of the information contained in the Assembly Parts Lists.

For instance, it can be seen either from Figure 2 or Figure 3 that each Article 7 or A_7 takes two A_5's directly. We use the word "directly" advisedly as A_7 requires, in total, four of A_5, as A_7 requires two A_1's directly and each A_1, in its turn, requires two of A_5 directly. We are, of course, interested to know how many of each article is required in total (directly or indirectly) for each other article. This information cannot be read directly from the Next Assembly Quantity Table, and our problem is how to determine these Total Requirement Factors from the Next Assembly Quantity Table.

* Note that we put "1's" in the diagonal. This makes the mathematical development easier.

Just the same way as we put the Next Assembly Quantities into a tabular form, we put the Total Requirement Factors into a table too.** To illustrate the case, Figure 4 shows the Total Requirement Factor Table associated with the illustrative example in Figure 2.

	1	2	3	4	5	6	7	8	9
1	1	3	0	3	0	0	1	1	1
2	0	1	0	0	0	0	0	0	0
3	7	33	1	27	3	0	13	10	10
4	0	0	0	1	0	0	0	0	0
5	2	10	0	8	1	0	4	3	3
6	2	12	0	8	1	1	4	3	6
7	0	1	0	1	0	0	1	0	0
8	0	2	0	0	0	0	0	1	1
9	0	0	0	0	0	0	0	0	1

Fig. 4. Total Requirement Factor Table. Observe, say, the *second* column relating to A_2. The *third* element from the top in this column relates to A_3 and displays the number 33. This means that 33 A_3's are required (in total) for each A_2.

We agree that each A_7 takes four of A_5's in total. In Figure 4, in the fifth row, under the seventh column, the number 4 is listed. The figure shows that 33 of A_3 is required for each A_2. Verification of this in Figure 2 requires a careful tracing of the various arrows. This tracing in Figure 2 can be described by the following statement:

Total number of A_5's required for each A_2 =

(Number of A_5's going directly into each A_1)

· (Total number of A_1's required for each A_2)

+ (Number of A_5's going directly into each A_7)

· (Total number of A_7's required for each A_2)

+ (Number of A_5's going directly into each A_8)

· (Total number of A_8's required for each A_2)

** Note that we put "O's" in the diagonal. This again makes the mathematical development easier.

Note again, carefully, the distinction between the statement "total number of A's required"...and "number of A's going directly into..."

We proceed now to put the above statement into mathematical form. We denote by $N_{i,p}$ the Next Assembly Quantity, which indicates the number of A_i's going directly into an A_p. Furthermore, we denote by $T_{p,j}$ the Total Requirement Factor, that is the total number of A_p's required for each A_j. Then the statement above can be written as:

$$T_{5,2} = N_{5,1} \cdot T_{1,2} N_{5,7} \cdot T_{7,2} N_{5,8} \cdot T_{8,2} \qquad (1)$$

or

$$T_{5,2} = \sum_p N_{5,p} T_{p,2} \qquad (2)$$

A schematic representation of these two equations is given in Figure 5.

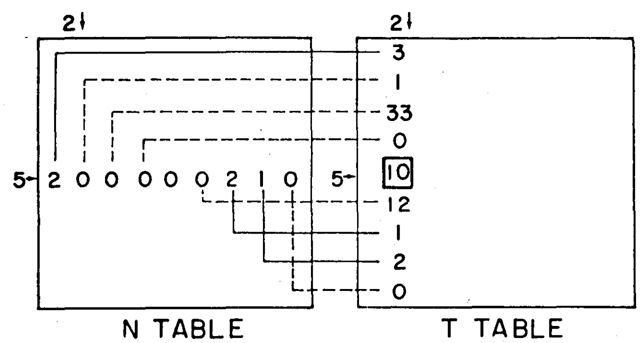

Fig. 5. Schematic representation of the equation

$$T_{5,2} = \sum_p N_{5,p} T_{p,2}$$

The second number of the fifth row of the T Table equals the "scaler multiple" of the fifth row of the N Table and second column of the T Table:

$$10 = 2 \times 3 + 0 \times 1 + 0 \times 33 + 0 \times 0 + 0 \times 10 + 0 \times 12$$
$$+ 2 \times 1 + 1 \times 2 + 0 \times 0$$

It is quite plausible now to believe that the above formula can be generalized for any pair of articles A_i and A_j; therefore, we write that

$$T_{i,j} = \sum_p N_{i,p} \cdot T_{p,j} \quad i \neq j \qquad (3)$$

Our convention about the "diagonal" elements can be written as

$$N_{i,i} = 1 \qquad (4)$$

$$T_{i,i} = 0 \qquad (5)$$

Equation 3 can be written in a more concise form by using *matrix algebra*:

$$[T] = \left[\frac{I}{I-N}\right] \qquad (6)$$

where $[I]$ denotes the unit matrix. It can be seen then, that our first problem, the problem of *the determination of the Total Requirement Factors, leads to a problem in matrix inversion, as shown by (6).* Or, to say it in another way, our problem is to solve the system of equations (3), (4) and (5) for the unknown T's. If these matrices had no special property, it would be practically impossible to carry through this computational task for thousands of articles. Fortunately, most of the elements on the matrices are zeros and the matrix is "triangular" and, consequently, special computational procedures can be developed.

At this stage, we have a formula for determining the parts required for each shippable article; the next problem of determining requirements to meet a given shipping schedule can be handled relatively easily. Suppose we are shipping only Articles A_2, A_4, A_9, and A_5, the last being a spare. How do we compute the quantity of A_5's required? Clearly

Quantity of A_5's required =

(Total number of A_5's required for each A_2)

x (shipping requirements of A_2)

+ (Total number ot A_5's required for each A_4)

x (shipping requirement of A_4)

+ (Total number of A_5's required for each A_9)

x (shipping requirement of A_9)

+ (Shipping requirement of A_5)

In order to put this in mathematical form, we introduce the notation that the shipping requirements are given by S_1, S_2,... and that the unknown requirements for the articles are X_1, X_2,... With this notation the above verbal statement can be written as

$$X_5 = T_{5,2} \cdot S_2 + T_{5,4} \cdot S_4 + T_{5,9} \cdot S_9 + S_5 \qquad (7)$$

This equation can again be written as

$$X_5 = \sum_k T_{5,k} \cdot S_k \qquad (8)$$

Again, it is plausible to generalize for any article A_i:

$$X_i = \sum_k T_{i,k} \cdot S_k \qquad (9)$$

This last equation then gives a method of determining the quantity of each article required, once the T matrix is known. It is important to recognize from the computational point of view that most of the S's are zero, as usually only a small fraction of the articles manufactured are shippable.

THE PROBLEM OF SCHEDULING

So far, we have concerned ourselves only with the problem of quantities required. Now we propose to introduce the time element. We will assume that our hypothetical factory operates in production periods, and we will assume that the shipping requirement is given, for each article, in each period. The question we prop ise to answer now is how many of each article is to be made in each period.

In every manufacturing process, articles must be made in a certain technological sequence. By inspecting Figure 2, one can see that, say, Article A_8 must be completed before A_9 can be started, as A_8 is required when A_9 is to be assembled. The question is, then, how long does it take to make A_9 and how much time should be allowed for making it.

Fig. 6 represents a routing sheet for manufacturing a certain article; the various operations, standard machine and labor hours required are shown. However, the time required from start to completion is not shown, as further inquiry into the method of manufacturing must be made, before this "make-span" can be determined.

Suppose first, that say, A_8 is manufactured on an assembly line. By a detailed analysis of the method of expending labor, and the time required between operations, the "assembly flow time" for A_8 can be determined. To this time one must add the time required to transport A_8 to the assembly line, which produces A_9; the total of these times is the "make-span" of A_9 and this is the time allowance that must be allowed when developing the production schedule.

PART NAME: SEAL STATIONARY FUEL NO. 53-12972

MATERIAL: NITRALLOY EZ OR "G" MODIFIED 1¼ RD SHEET NO. 1 OF 1

OPERATION	OPERATION NO	DEPARTMENT AND GROUP	CODE	STANDARD HOURS PER 100 MAN		MACH.	
FACE, DRILL, REAM, ETC.	10-4270	301-45	1002	1	11 H	2	22
COPPER PLATE	13-			PURCHASED			
DEGREASE	14-			PURCHASED			
DRAW TO SPEC 501	16-			PURCHASED			
RECEIVE							
HARDNESS INSPECT	25-6900	314-89	5508		20 H		
ROUGH GRIND ETC.	30-2840	305-52	2404		13 H		
NITRIDE TO SPEC #1600	55-			PURCHASED			
FINISH GRIND FLANGE SIDE	65-2840	305-52	2404		15 H		
MAGNETIC PARTICLE INSPECT	90-6900	314-85	5612		42 H		
DEMAGNETIZE	112-6900	314-89	5508		01 H		
STOCK		362-					

Fig. 6. Typical Factory Routing Sheet

On the other hand, suppose that A_8 is manufactured in a job shop; that is A_8 is manufactured in various lots, at various times. What should the make-span of A_8 be in such a case? It turns out that here the make-spans primarily depend on the number of operations required, as most of the time is spent by the lot "waiting" for the next operation. Such a situation is shown in Fig. 7, where time required is plotted against number of operations.

In both cases, it is convenient to assume that there is such a thing as a *make-span* associated with each article. By combining these make-spans, it is possible to establish set-back charts as illustrated in Fig. 8. It can be seen, for instance, that A_9 is allowed eight production periods, 5 periods of manufacturing A_9, and 3 production periods as a safety factor or "cushion." The set-back chart tells us that, say, A_5 must be started 19 production periods prior to the shipping day of A_9. If, say, 100 of A_9 must be shipped in the 30th production period, then 3(100) of A_5 must be made in the 11th production period, as each A_9 requires a total of 3 A_5's.

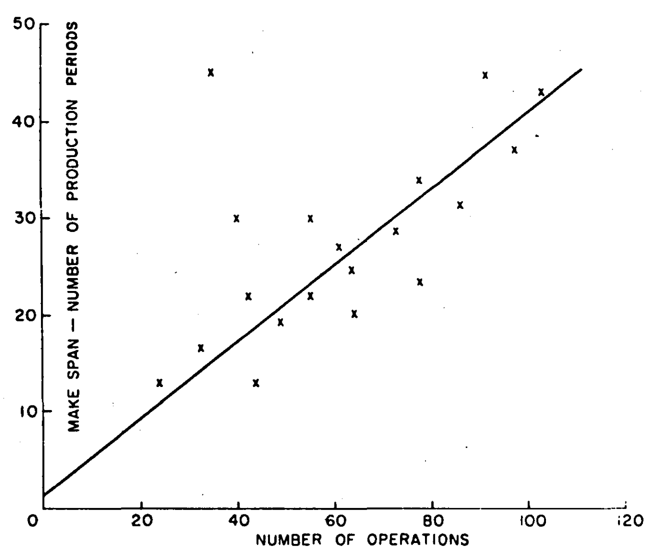

Fig. 7. Statistical Relationship between Number of Operations and Make Span.

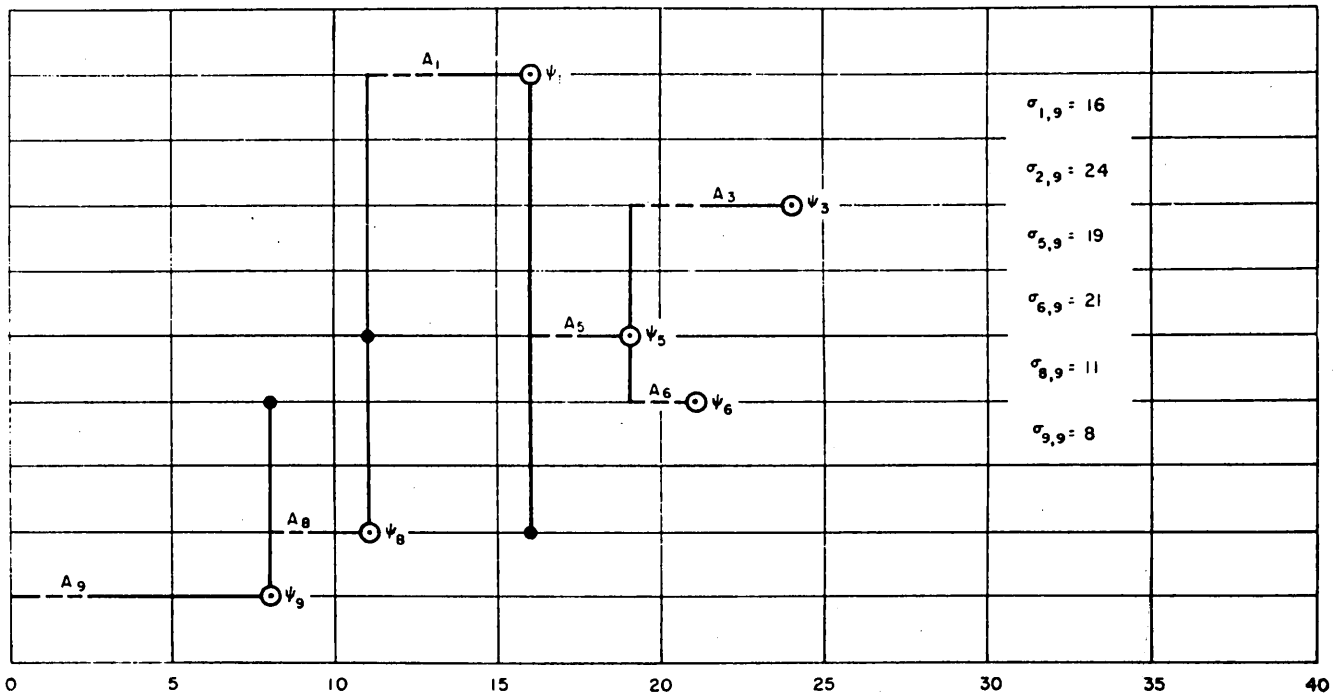

Fig. 8. Setback Chart in Assembly A_9 as an Example, the Setback of A_1 is 16.

However, this is not all: A_5's are also required for shipping A_2's and A_4's. If A_1, A_7, and A_8 are shipped as spares, there will be additional requirements for A_5. In order to handle this problem, we assume that a complete setback chart is developed "in" each article (Fig. 8 is the setback chart "in" A_9) and that a setback matrix, as shown in Fig. 9 is developed, where each column represents a setback chart. This table in Fig. 9 shows, for instance, that in order to ship A_2's, (second column), A_5's (fifth row) must be made in advance of 18 periods.

Suppose now that the shipping schedules are given in tabular form as shown in the upper part of Fig. 10, where each row refers to an article and each column to a production period. Suppose that the manufacturing schedules are to be developed as shown on the lower part of the figure. Let s_k^m (the number in the k'th row and m'th column) denote the number of A_k's to be shipped in the m'th period and let x_k^m denote the quantity of A_k to be manufactured in the same period. Then, for instance,

$$x_5^{11} = 2s_1^{19} + 10s_2^{29} + 8s_y^{30} + s_5^{14} + 4s_7^{24} + 3s_8^{22} + 3s_9^{30} \qquad (10)$$

	1	2	3	4	5	6	7	8	9
1	5	15	0	16	0	0	10	8	16
2	0	7	0	0	0	0	0	0	0
3	13	23	0	24	8	0	18	16	24
4	0	0	0	6	0	0	0	0	0
5	8	18	0	19	3	0	13	11	19
6	10	20	0	21	5	0	15	13	21
7	0	12	0	11	0	0	5	0	0
8	0	10	0	0	0	0	0	3	11
9	0	0	0	0	0	0	0	0	8

Fig. 9. Setback Matrix $[\sigma]$. Each column represents a setback chart. For instance, the ninth column represents the setback chart shown in Figure 8.

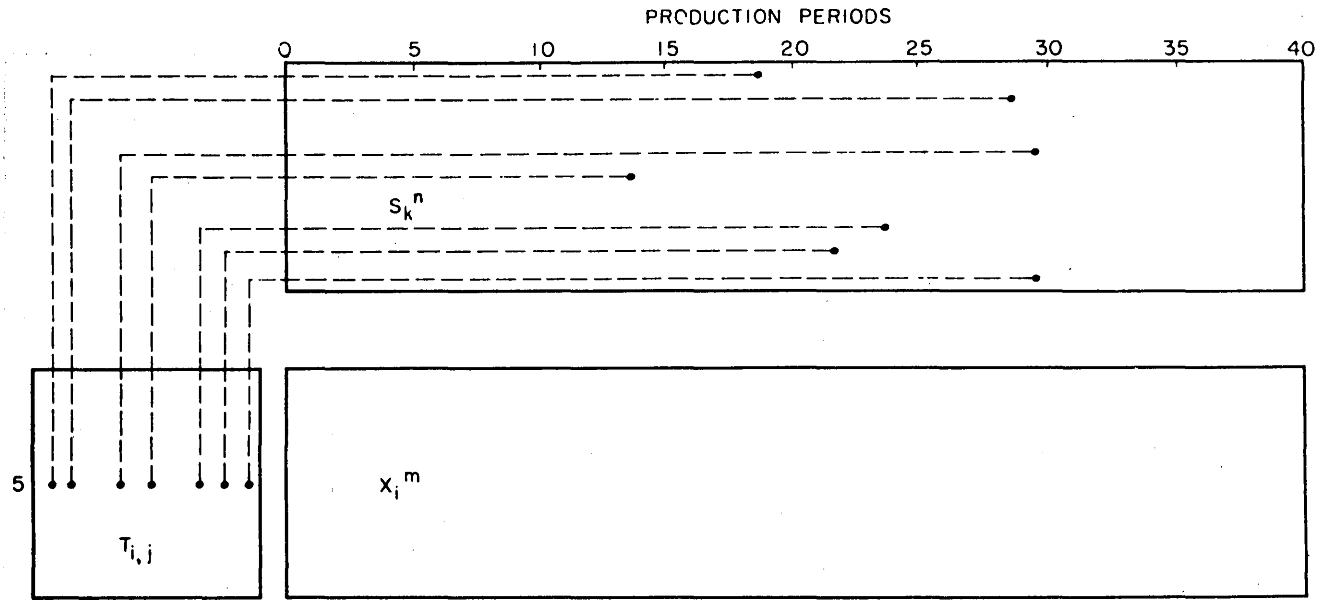

Fig. 10. Representation of the Scheduling Equation.

$$x_5^{11} = T_{5,1}\, s_1^{19} + T_{5,2}\, s_2^{29} + T_{5,4}\, s_4^{30} + T_{5,5}\, s_5^{14}$$

$$+ T_{5,7}\, s_7^{24} + T_{5,8}\, s_8^{22} + T_{5,9}\, s_9^{30}$$

as shown schematically in Fig. 10. A somewhat simpler schematic representation is shown in Fig. 11, where the various shipping schedules are transposed or set-back so that the method of computation becomes *matrix multiplication.*

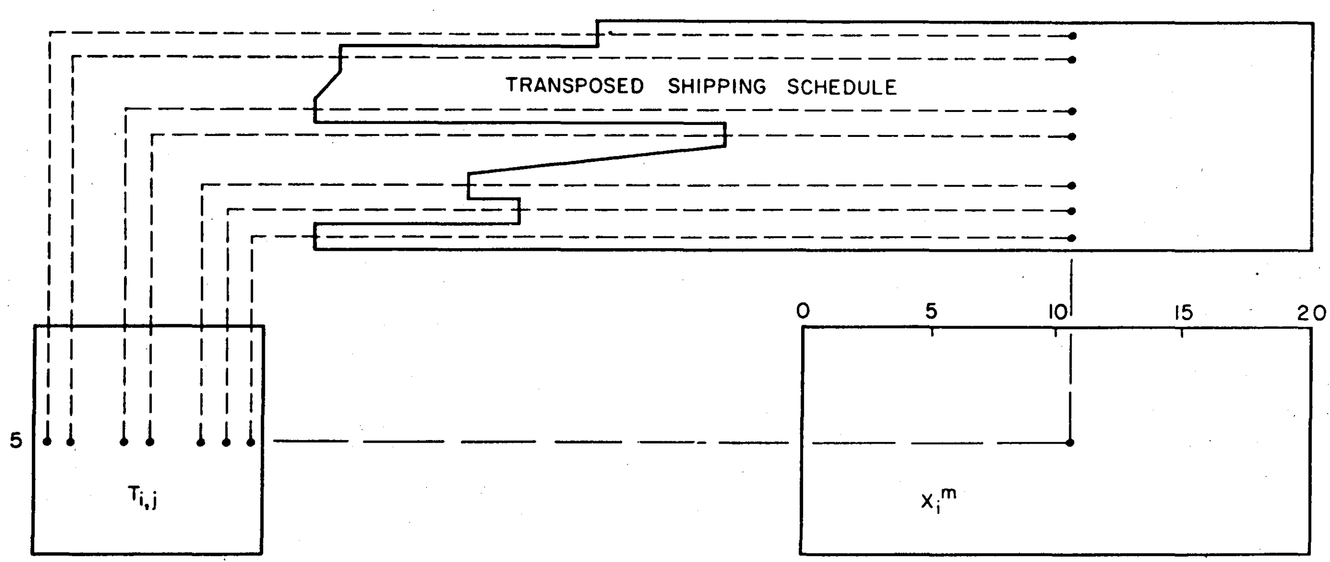

Fig. 11. Scheduling as a Problem in Matrix Multiplication.

Let us denote by $o_{i,j}$ the set-back of A_i *in* A_j ; then this number is shown in the i'th row and j'th column of Fig. 9. With this notation, equation (10) can be written as

$$x_5^{11} = 2s_1^{11+\sigma_{5,1}} + 10s_2^{11+\sigma_{5,2}} + 8s_3^{11+\sigma_{5,4}} + s_5^{11+\sigma_{5,5}} + 4s_7^{11+\sigma_{5,7}} + 3s_8^{11+\sigma_{5,8}} + 3s_9^{11+\sigma_{5,9}} \qquad (11)$$

This same equation holds not only for the 11th period, but for any production period m and so we can write

$$x_5^m = \sum_k T_{5,k}\, s_k^{m+\sigma_{5,k}} \qquad (12)$$

Again, the equation holds not only for A_5, but for any article A_i:

$$x_i^m = \sum_k T_{i,k}\, s_k^{m+\sigma_{i,k}} \qquad (13)$$

This last equation is our scheduling equation: *it relates the unknown number of articles to be manufactured to the given shipping schedule.*

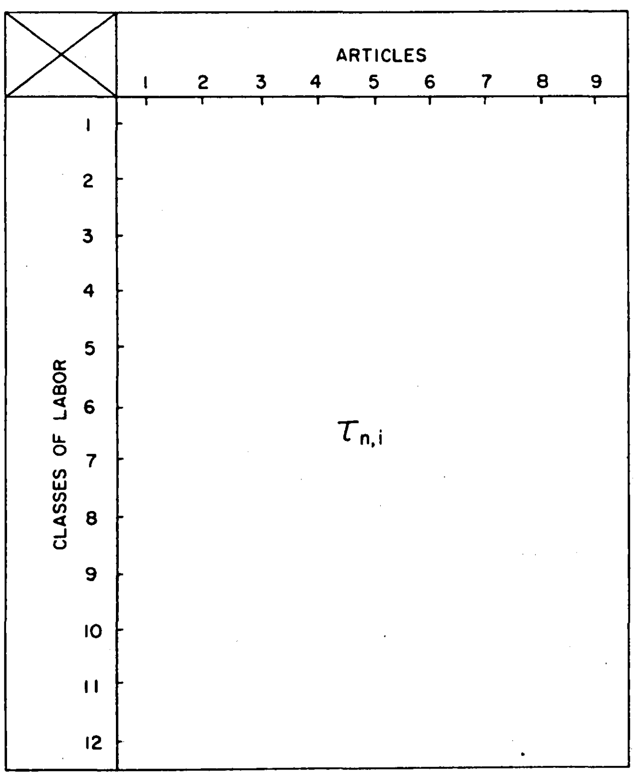

Fig. 12. Manhours Requirement Matrix.

MACHINE LOADING

Finally, we proceed to the machine hours computations associated with the schedule. We assume that each article is routed through a number of machines and that the standard times are available from routing sheets similar to Fig. 6. This information is to be represented as a $\tau_{n,i}$ matrix shown in Fig. 12, where $\tau_{n,i}$ denotes the number of hours required on machine type n, to manufactured article A_i.

The total labor hours required in production period m, on machine type n, is given by the *matrix multiplication* relationship

$$h_n^m = \sum_i \tau_{n,i} x_i^m \qquad (14)$$

as shown schematically in Fig. 13. When equation (14) is combined with equation (13) we get

$$h_n^m = \sum_i \sum_k T_{i,k}\, s_k^{m+\sigma_{i,k}} \qquad (15)$$

This last equation relates the machine hours associated with the given shipping schedule.

In these formulae we neglected set-up time. If this is important, the formula can be modified by adding these set-up times. Man hour computations for assembly or other type of labor can be computed in a similar fashion.

In summary, then, we see that if a shipping schedule is given, equations (3), (9) and (13) allow the computation of the number of articles that must be manufactured in each production period; equations (14) or (15) allow determination of the machine hour requirements in these production periods. *All these equations are formulae from matrix algebra.*

ADVANTAGES OF USING MATHEMATICAL MODEL

The mathematical model developed in this paper describes the problem of parts listing and scheduling (or at least some aspect of the problem) better than a verbal description. The concepts used in the mathematical model, such as the next assembly quantity, total requirement factor, the set-back time, are all concepts currently used in industry. It is believed, though, that setting these concepts into mathematical form and stating their inter-relationship mathematically greatly contributes to the clear understanding of these concepts, and also to the comprehension of their quantitative inter-relationships.

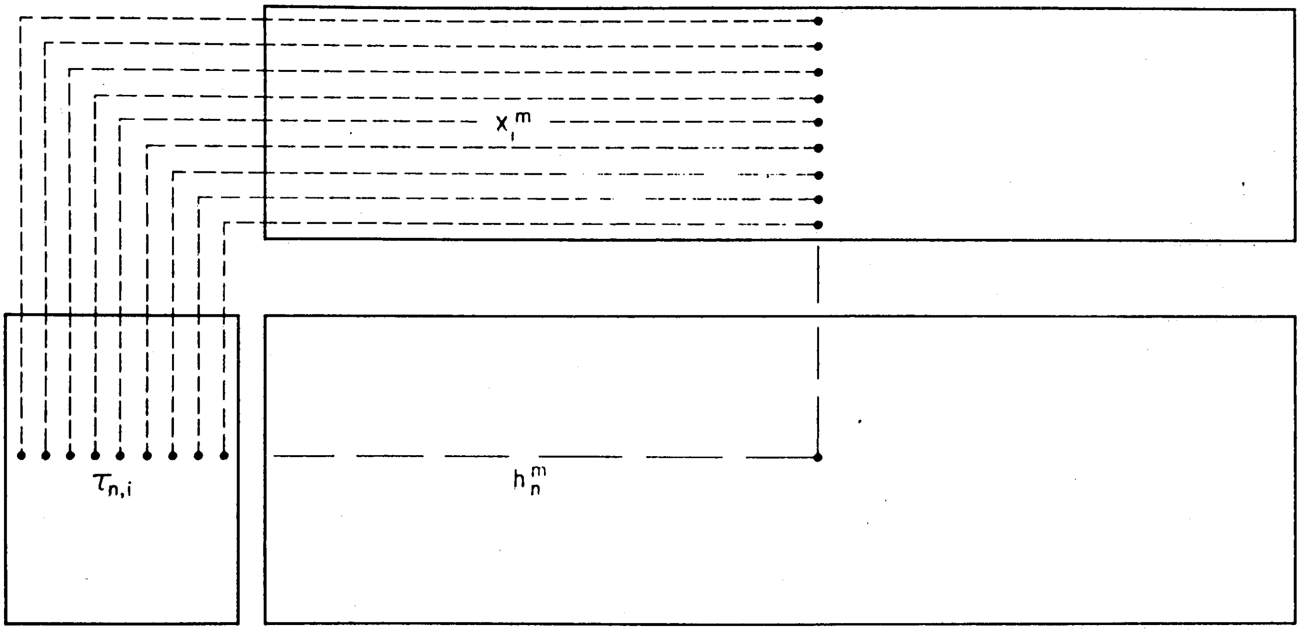

Fig. 13. Scheduling of Labor with Matrix Multiplication:

$$h_n^m = \sum_i \tau_{n,i}\, x_i^m$$

We proceed now to describe the advantages to be gained from the use of mathematical models in this or other management fields:

1. A mathematical model indicates what data should be collected to deal with the problem quantitatively.

In our particular case, for instance, the concept of set backs is brought into focus. From the presentation you might think that this is quite a straightforward concept, and used by all manufacturers. The fact of the matter is that the concept is not usually used in a very clear-cut fashion. Different people in different plants, or even within the same plant, have a different notion of what these set-backs are. Such vagueness in concepts leads to a great deal of frustration and lost motion. The advantages to be gained by a common understanding as imposed by the mathematical model, leads to significant advantages in production control.

2. A mathematical model establishes indices of effectiveness and control.

We can again refer in this particular case to the concept of set-backs. Once this concept is firmly understood it leads to a method of control of a particular production department. As time goes on one can determine whether this particular department is improving or getting worse. Or, again, it is possible to compare various departments and see how their particular effectiveness compares.

3. The mathematical model makes it possible to use mathematical techniques that otherwise appear to have no applicability to the problem.

Who would have thought that matrix algebra is applicable to parts listing? The advantage to be gained by using a known mathematical technique is not to be ignored. We have found that when the detailed computational techniques for parts listing and scheduling are developed, known mathematical techniques from matrix algebra become of importance.

4. The mathematical model makes it possible to deal with the problem in its entirety and allows a consideration of all the variables of the problem simultaneously.

In our particular case, relationships were determined between the shipping schedule, the number of parts manufactured in each production period, and the labor loads imposed by this particular shipping schedule. If any of these production variables are changed, our equations permit the determination of the effect of these changes on the other production variables.

5. The mathematical model is capable of being enlarged, step by step to a more comprehensive model, to include factors that are neglected in verbal descriptions.

There are many important factors that are left out of our mathematical model. For instance, we have not talked about inventory levels, accounts payable or accounts receivable. Let us stress here that the mathematical model presented here is only a beginning. We have found, however, that when dealing with specific problems it is possible to include these other factors by modifications of the mathematical model.

6. The mathematical model uncovers relations between the various aspects of the problem which are not apparent in the verbal description.

When we developed this mathematical model further, we discovered relationships between inventory levels and tightness of schedules. When set-backs are made large, large inventories follow, but the plant manager finds it easier to keep the schedule. On the other hand, when setbacks are made short, inventory levels go down, but effort and expense must be expended in keeping this tight schedule.

7. The mathematical model prepares the groundwork for the introduction of large-scale electronic data processors.

The scheduling of manufacturing operations require the processing of large amounts of data. It is expected that in the future large-scale electronic data processors will be able to handle the bulk of this data processing, and, therefore, that electronic computers will play a prominent role in production control. Manufacturing firms making plans for the introduction of electronic computers are experiencing considerable difficulty in introducing these computers effectively. Electronic computers by their nature are machines that compute and perform logical operations, and, consequently, the problem of obtaining solutions through electronic computers can be greatly alleviated by the introduction of mathematical models.

8. The mathematical model frequently leads to a solution that can be adequately described and justified on the basis of a verbal description.

The mathematical model described in this paper uses such concepts as the next assembly quantity matrix, total requirement factor matrix, set-back matrix, etc. This does not mean, however, that the various operating personnel dealing with these variables, must learn the mathematics of matrix algebra. These people usually deal only with certain zones of the problem, and it is usually possible to describe these various zones without using mathematical equations.

Let me thank you now for bearing with me—voluntarily, or involuntarily—in developing this long argument in favor of the use of mathematical models in management and business. As a final note let me point out that mathematical models perhaps should be compared with maps. Most of us use maps to find our way around when we travel. However, the multitudes of problems that beset us while traveling are not usually blamed on the maps we use. Mathematical models will become more and more important in business; they will help to describe and to solve management problems. However, a mathematical model does not necessarily include all the facts of a business situation, and we should not plan to obtain solutions to management problems by "formulas."