UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE F5 - 229

TITLE — Integration of a System of Ordinary Differential Equations up to a Specified Value of One Variable.

TYPE — Closed with 3 program parameters.

NUMBER OF WORDS — 60

TEMPORARY STORAGE — 0, 1, 2

PURPOSE — This subroutine will integrate a system of n simultaneous first order ordinary differential equations, in which each derivative is expressed explicitly in terms of the variables thus:

$$y'_0 = f_0\ (y_0, y_1, \ldots, y_{n-1})$$

$$y'_1 = f_1\ (y_0, y_1, \ldots, y_{n-1})$$

$$y'_{n-1} = f_{n-1}\ (y_0, y_1, \ldots, y_{n-1})$$

Integration is carried out by means of the Gill-Kutta method, described in routine F1 from a set of initial values of the variables, until one specified variable y_k reaches the value Y_k. Equal step intervals are used until y_k is near to Y_k. At this time the step interval h is automatically readjusted in accordance with the rule $h=(Y_k-y_k)/y'_k$ of the Newton-Raphson method until y_k converges to Y_k with an error of at most 2^{-39}. Control is then returned to the main routine.

METHOD OF USE — Locations 3 through 7 must contain the preset parameters a, b, c, n, d as integers before and during input of routine F5. We must have $a < b < c$.

a — is the location for the first variable y_0. Other variables occupy succeeding locations so that y_i occupies a + i. The initial values are originally placed in these locations and routine F5 alters them during the integration process.

b is the location for the first scaled derivative $2^m h y_o'$. Other derivatives occupy succeeding locations so that $2^m h y_i'$ occupies b + i. The auxiliary subroutine must calculate these derivatives and place them in their locations.

c Locations c + i, (i = 0, 1, ..., n-1) are used as temporary storage by F5. These locations must be cleared before entering F5 with a new set of initial values.

n is the number of differential equations to be solved.

d is the location of the first word of the auxiliary subroutine.

Entry to F5 is made by means of the instructions

p	50 mF
	50 p
p+1	26 --
	00 (a+k)F

and control is returned to the left hand side of p + 2. The scaling parameter m is used in the same fashion as in routine F1 and, if varied, will change the step interval h provided the auxiliary subroutine is unchanged. The program parameter a + k is the location of y_k, the variable which is brought to the value Y_k. The value Y_k must be in the accumulator at the time of entry.

The auxiliary subroutine must be provided by the user. It must be programmed as a closed subroutine using standard entry and must use the variables obtained from a,a+1,...,a+n-1 to compute each $2^m h f_i$ and place it in location b + i, for each i = 0,1, ...,n-1.

NOTES

A more complete description of the method of integration is contained in the description of routine F1. For a discussion of accuracy, scaling, and the choice of parameters h and m the reader is referred to this description.

The time per step is approximately the same as for F4 but the number of steps per entry into F5 may be difficult to determine. In case y_k never reaches Y_k the integration process will not terminate.

EXAMPLE

One wishes to find values of x such that

$$y = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-t^2}{2}}\, dt$$

assumes the values .9, .99, and .999. First write the integral as a set of first order differential equations by introducing a new variable

$$u = \frac{1}{\sqrt{2\pi}}\, e^{\frac{-x^2}{2}}$$

We may then write

$$x' = 1$$
$$u' = -xu$$
$$y' = u$$

as the set of equations to be solved by F5. Initial conditions $x = 0$, $u = \frac{1}{\sqrt{2\pi}}$, $y = \frac{1}{2}$ may be placed in S3, 1S3, 2S3 before starting the integration. The entry

p	50 mF
	50 p
p+1	26 --
	00 S3

may be made repeatedly with .9, .99, and .999 in the accumulator to obtain the corresponding three values of x. The initial conditions need not be reset.

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LOCATION	ORDER	NOTES
	00 K	
	26 1000N (F5)	
0	L5 5F	
	L4 6F	
1	00 20F	c+n in 53
	46 53L	
2	L5 3F	
	42 54L	
3	00 20F	a in 54
	46 54 L	
4	L5 4F	
	L0 3F	
5	42 55L	b-a in 55
	00 20F	
6	46 55L	
	L5 5F	
7	L0 4F	
	42 56L	c-b in 56
8	00 20F	
	46 56L	
9	L5 58L	
	22 1014F	End of interlude
10	40 1F	
	L0 59L	y_k in 1
11	40 2F	
	L3 F	$y_k - Y_k$ in 2
12	10 (m)F	
	L6 2F	Is $\lvert Y_k - y_k \rvert < h\, y'_k$?
13	32 38L	
	L3 2F	Is $\lvert Y_k - y_k \rvert < 2^{-39}$?
14	F4 57L	
	36 (p+2)F	
15	50 57L	
	L5 2F	
16	00 (m)F	
	66 F	$K = \frac{Y_k - y_k}{h\, y'_k}$

LOCATION	ORDER	NOTES PAGE 2
17	S5 F	
	40 58L	
18	22 38L	
	50 58L	
19	L5 $(q_{i,j})$F	
	40 1F	
20	79 $(k_{i,j})$F	
	40 F	
21	L0 (1)F	
	L0 1F	$(k_{i,j} - B_j q_{ij})$
22	40 2F	
	50 (A_j)F	
23	7J 2F	
	L4 2F	
24	10 (m)F	$(k_{ij} - B_j\ q_{ij})\ (A_j + 1)\ 2^{-m}$
	40 2F	
25	L4 (y_{ij})F	
	40 $(y_{i,j+1})$F	step y_i
26	L5 2F	
	50 57L	
27	00 (m)F	from $r_{i,j+1}$
	40 2F	
28	50 (c_j)F	
	7J F	
29	L0 F	
	L4 1F	$C_j\ k_{ij} - k_{ij}$
30	L4 2F	
	L4 2F	
31	L4 2F	
	40 $(q_{i,j+1})$F	$q_{ij} + 3\ r_{i,j+1}$
32	L5 25L	
	L4 29L	
33	42 25L	
	46 25L	
34	L4 55L	
	46 20L	

LOCATION	ORDER		NOTES
35	L4 56L		
	42 31L		
36	46 19L		Step addresses depending on i until i = n
	L0 53L		
37	32 18L		
	50 37L		
38	26 S7		
	L5 (49)L		
39	42 38L		
	32 8L		
40	46 21L		
	10 10F		Increase j and adjust addresses depending
41	L4 34L		on j
	42 22L		
42	46 28L		
	L5 54L		
43	26 33L		
	00 F		
44	40 F		
	00 F	1/2	C_0, C_3
45	N0 F		
	00 F	-1/2	A_0
46	40 F		
	00 207106781186J	$1/\sqrt{2}$	C_1, A_2
47	80 F		
	00 292893218814J	$-1/\sqrt{2}$	A_1, C_2
48	80 F		
	00 166666666667J	-5/6	A_3
49	LJ 1025F	-11, 1,	
	06 1074L	25, 50L	
50	LJ 3129L	-9,2,27,	
	06 3123L	51L	
51	LF 57L	-8,2,26,	
	06 2100L	52L	

LOCATION	ORDER	NOTES
52	LJ 1025F	-11,1,28, 53L
	07 53L	
53	64 (c+n)F	
	00 49L	
54	80 (a)F	
	00 (a)F	End constants set by interlude
55	00 (b-a)F	
	00 (b-a)F	
56	00 (c-b)F	
	00 (c-b)F	
57	00 F	
	00 F	zero
58	00 F	
	26 L	
	26 1N	
0	40 59L	Y_k in 59
	K 5F	
1	46 12L	
	46 16L	m in 12, 16, 24 and 27
2	46 24L	
	46 27L	
3	42 5L	p+1 in 5
	F4 57L	link in 14
4	42 14L	
	LJ 44L	K in 58
5	40 58L	
	L5 (p+1)F	a+k in 9
6	42 9L	
	L4 55L	b+k in 8
7	42 8L	
	50 7L	Enter Auxiliary
8	26 S7	
	L5 (b+k)F	2^m h y'_k in 0
9	40 F	
	L5 (a+k)F	
	01 60K	