

## UNIVERSITY OF ILLINOIS

## DIGITAL COMPUTER

ILLINOIS CODE L3 - 100

TITLE The Complete Linear Equation Solver (DOI Only)

TYPE Entire Program

ACCURACY Depends upon the condition of the equations to be solved and the number of iterations carried out.

DURATION (a) - about 15 seconds to input program.  
 (b) - about  $n^2/20$  (resp.  $n^2/100$ ) seconds for each input of coefficients if coefficients are punched to 12 (resp. 2) figures;  $n$  is the number of equations being solved.  
 (c) - about  $n^3/1400$  seconds to solve equations for each iteration.  
 (d) - about  $2n/5$  seconds to punch results on each iteration (to 11 figures, the maximum allowable)  
 (e) - when  $n = 37$  (the maximum allowable) the time is about 4 minutes.

PUNCHING DATA To solve the set of equations

$$\sum_{j=0}^{n-1} a_{ij} x_j + a_{in} = 0$$

we proceed as follows:

(a) Scale the coefficients (usually simply by moving the decimal point) so that each coefficient is less than  $1/2$ .

(b) Punch each scaled coefficient as a decimal with up to 12 places preceded by a sign, e.g., +016, -204, the decimal point being considered to follow the sign. (On some teletype keyboards + = K, - = S).

(c) Terminate each row  $a_{i0}, \dots, a_{in}$  of coefficients by punching the character N.

(d) Follow the last N by a sexadecimal character p which determines the number of decimal digits to be printed in the results. The character p can assume the values 1, 2,

3, ..., 9, K, or S where K = 10 and S = 11. Spaces (5 holes) may be punched at will in the data tape, and a group of spaces should precede the first coefficient.

**MATHEMATICAL METHOD USED:**

Let  $x_0, x_1, \dots, x_{n-1}, x_n$  be the machine representations of the correct solutions to the set of equations

$$\sum_{j=0}^{n-1} a_{ij} x_j + a_{in} x_n = 0; \quad i = 0, 1, \dots, n-1$$

( $x_n$  is a scale factor such that the numerical values  $(x_i)_{\text{num}}$  of the solutions are

$$(x_i)_{\text{num}} = x_i / x_n, \quad i = 0, 1, \dots, n-1)$$

If a superscript  $k$  characterizes the set of solutions at the  $k^{\text{th}}$  stage of iteration, then

$$\sum_{j=0}^{n-1} a_{ij} x_j^{(k)} + a_{in} x_n^{(0)} \equiv \delta_i^{(k)} x_n^{(0)}$$

defines the residue at the  $k^{\text{th}}$  stage. (The scale factor  $x_n^{(k)}$  is forced to remain  $x_n^{(0)}$ ). If we define  $\epsilon_i^{(k)}$ , corrections to the solutions at the  $k^{\text{th}}$  stage, as:

$$x_i \equiv \epsilon_i^{(k)} + x_i^{(k)}$$

then the  $\epsilon$ 's are the solutions of:

$$\sum_{j=0}^{n-1} a_{ij} \epsilon_j^{(k)} + \delta_i^{(k)} x_n^{(0)} = 0 \quad (1)$$

If these equations be solved, we obtain a better approximation to our correct solutions as:

$$x_i^{(k+1)} \equiv \epsilon_i^{(k)} + x_i^{(k)} \quad (2)$$

The process is then repeated ad libitum. Instead of using equations (1) and (2) we use

$$\sum_{j=0}^{n-1} a_{ij} \epsilon_j^{(k)} + \delta_i^{(k)} x_n^{(0)} B = 0, \quad (1a)$$

$$x_1^{(k+1)} = B^{-1} \epsilon_1^{(k)} + x_1^{(k)}, \quad (2a)$$

where  $B = 2^{6+4k}$ ;  $k = 0, 1, 2, \dots$ ,

Appropriate adjustments of scale factor are made and all numbers are tested for exceeding capacity in which case B is appropriately reduced.

#### METHOD OF USE

Case I: Problem placed on machine for first time, i.e., we have as yet no approximate solutions of any kind.

- 1 - Place program in reader and start with white switch.
- 2 - Bypass the stop that will occur about one-third of the way in by using black switch.
- 3 - When program is in, place coefficient tape in reader and start with black switch.
- 4 - Coefficient tape will be devoured and  $x_1^{(0)}$  tape will be punched out; the machine will stop on an 80028 24025 order (sexadecimal).
- 5 - Rewind the coefficient tape and replace in reader. Upon starting (with black switch) the machine will punch out  $x_1^{(1)}$  and stop on the 80028 24025 order.
- 6 - The above step may be repeated as often as desired producing  $x_1^{(k)}$ .
- 7 - If, during read-in of the coefficient tape, the machine should stop on a 24025 15000 order (sexadecimal) before the tape is totally in, rewind tape and place in reader again. Start the machine again (using black switch). The process will be repeated using a new factor B, reduced from the last previous one by a factor  $2^4$ .
- 8 - Step 7 can be repeated until the coefficient tape is read-in. If, in this process, the machine ever hangs up on a 19000 12000 order (sexadecimal) from memory location OKN (sexadecimal), we have an indication that the equations are so badly conditioned that their residues will not accept a multiplication by any  $B > 1$  without exceeding capacity. The programmer is advised to go elsewhere for his solution.

Case II: A continuation of iteration after any previous stage and after machine has been cleared.

- 1 - If after leaving the machine and examining his results at leisure the programmer desires several more iterations, the problem does not have to be repeated from the beginning ( $k=0$ ).
- 2 - Read program into machine with white switch. When tape stops, remove it from the reader and move about three feet to the left setting it in the reader on the large group of spaces found there. Start machine (black switch) and read rest of program into machine.
- 3 - Place any previous answer tape containing  $x_i^{(k)}$  in reader (on initial group of spaces) and start machine (black switch). This tape will be read into the machine.
- 4 - When machine stops, place coefficient tape in reader and start machine. At this point we are at the same stage as in step 3 above.  $x_i^{(k+1)}$  will be produced, etc.

FORM OF RESULTS

- 1 - The results will be printed in a column with an extra space indicating the position of the decimal point. The last entry is the scale factor  $x_n^{(0)}$  mentioned above. For example, if we print

12	04	the results are --	12.04
03	72		3.72
00	17		.17
01	00		

- 2 - The above column of results will be followed by 20 sexadecimal digits which needn't be printed but which are necessary on subsequent read-ins. The 20 digits are:

J2 a b c 500LJ10 d e f 66 g h i

where (a b c) = (10p + q) where  $10^{-q}$  is the scale factor  $x_n^{(0)}$  and p is the number of digits printed.

(d e f) = r where  $2^r = B$ , the factor to be used on the next iteration;

(g h i) = (246 + n) where n is the number of equations being solved.

NOTE

Suppose  $k^{\text{th}}$  iterated solutions  $x_i^{(k)}$  are obtained with Method I carried to the  $k^{\text{th}}$  stage and by Method II by inserting the  $(k-1)^{\text{th}}$  solutions. These two will not agree exactly since iterations are performed in Method I using 12 place previous results stored in the machine, while the  $(k-1)^{\text{th}}$  stage results with which Method II starts can be punched out to only 11 figures.

RT: 5/20/59

DATE <u>November 1, 1956</u>
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lgr

LOCATION	ORDER	NOTES	PAGE 1
	DECIMAL ORDER INPUT		
	00 3K		
3	40 208F	Put $x_0^{(2)}, \dots, x_n^{(2)}$ in $0, \dots, n$	
	50 3F		
4	26 136F		
	L5 144F	$(\lambda_{n+1})$ in $R_1$ (L.H.)	
5	10 20F	$(\lambda_{n+1})$ in $R_1$ (R.H.)	
	42 132F		
6	42 114F		
	L0 104F	$(\lambda_n)$ in $R_1$ (R.H.)	
7	42 133F		
	42 123F		
8	42 170F		
	L0 131F	$(n)$ in $R_1$ (R.H.)	
9	42 109F		
	00 20F	$(n)$ in $R_1$ (L.H.)	
10	46 109F		
	L4 203F	$(\lambda_n)$ in $R_1$ (L.H.)	
11	46 179F		
	L5 106F		
12	L4 109F	-- $(r_n)$ -- $(r_n)$ in $R_1$	
	46 178F		
13	42 179F		
	46 82F		
14	42 90F		
	42 124F		
15	46 133F		
	L4 104F	-- $(r_{n+1})$ -- $(r_{n+1})$ in $R_1$	
16	46 105F		
	42 105F		
17	46 107F		
	46 132F		
18	L0 143F	$(r_n - 1)$ in $R_1$ (L.H.)	
	46 61F		
19	L5 191F		
	42 100F		

LOCATION	ORDER	NOTES
20	L5 141F 42 137F	Set switches for residue computation
21	L5 155F 42 147F	
22	81 36F 46 29F	Read proper proper print parameter from tape and set in printer
23	L5 122F 46 91F	Open print adjusting routine switch
24	81 40F 46 172F	Set shift orders.
25	46 177F 46 124F	
26	24 37F L5 131F	Stop for tape, then compute correction Set initial address in printer
27	42 28F 92 1017F	From 206 15 spaces
28	92 135F L5 ( $\lambda_0$ )F	3 line feeds by 27,32
29	J2 ( )F 50 29F	By 22 Print $x_0, \dots, x_n$ from $\lambda_0, \dots, \lambda_n$
30	26 180F 92 129F	
31	L5 28F L4 104F	
32	42 28F L0 114F	
33	32 28F L5 29F	Put print order on tape
34	82 40F L5 124F	Put shift order on tape
35	82 40F 24 37F	Stop for tape then compute correction
36	00 F 00 F 24 999N	

LOCATION	ORDER	NOTES	PAGE 3
The following will overwrite the above if the last stop is bypassed.			
3	00 3K	Read in first row of matrix	
4	40 246F		
	50 3F		
4	26 136F		
	L5 144F	$(r_{n+1})$ in $R_1$ (L.H.)	
5	46 105F		
	46 107F		
6	46 132F		
	10 20F	$(r_{n+1})$ in $R_1$ (R.H.)	
7	42 105F		
	L5 105F		
8	L0 104F	-- $(r_n)$ -- $(r_n)$ in $R_1$	
	46 133F		
9	46 178F		
	46 82F		
10	42 179F		
	42 90F		
11	42 124F		
	L0 104F	-- $(\nu_n - 1)$ -- $(\nu_n - 1)$ in $R_1$	
12	46 61F		
	L0 207F	-- $(n)$ -- $(n)$ in $R_1$	
13	46 109F		
	42 109F		
14	L4 203F	$\lambda_n$ in $R_1$ (L.H.)	
	46 179F		
15	10 20F	$\lambda_n$ in $R_1$ (R.H.)	
	42 170F		
16	42 123F		
	42 133F		
17	L4 104F		
	42 114F	$(\lambda_{n+1})$ in $R_1$ (R.H.)	
18	42 132F		
	41 3F		



LOCATION	ORDER	NOTES	PAGE 4
19	L5 104F	Set print parameter to 1	
	46 29F		
20	22 40F	Go to code 47	
	81 4F	From 119	
21	10 39F	Compute printing parameter	
	75 161F		
22	00 59F		
	L4 29F		
23	46 29F		
	L5 191F	Set switches for subsequent computation	
24	42 100F	of residues	
	L5 141F		
25	42 137F		
	L5 155F		
26	42 147F		
	L5 131F	From 206	
27	42 28F	Set initial address in printer	
	92 1017F	15 spaces	
28	92 135F	3 line feeds	
	L5 ( $\lambda_0$ )F	By 27,32	
29	J2 ( )F	By 19,29	
	50 29F	135	
30	26 180F	Print $x_0, \dots, x_n$ from $\lambda_0, \dots, \lambda_n$	
	92 129F	Line Feed	
31	L5 28F		
	L4 104F		
32	42 28F		
	L0 114F		
33	32 28F		
	L5 29F	Put print order on tape	
34	82 40F		
	L5 122F	Open print adjusting routine switch	
35	46 91F		
	L5 124F	Print number of shifts	
36	82 40F		
	24 37F	Stop for tape, then compute correction	

LOCATION	ORDER		NOTES	PAGE 5
	00 37K			
37 (0)	41 3F	From 35F	$m = 0$	
	L5 1L		to set $M(a_{00}) = 0$	
38(1)	42 22L	By 44	Address is used.	
	50 284F			
39 (2)	40 246F			
	50 2L		Input row $m = r_0, \dots, r_n = a_{mi}$	
40 (3)	26 136F		$i = 0, \dots, n$	
	41 4F	From 20F	$i = 0$	
41 (4)	L5 22L			
	42 5L		Make $a_{m0} = 0$ so that the virtual $n$ th	
42 (5)	49 2F		row will be interchanged with the	
	41(a <sub>10</sub> )F	By 4	input row.	
43(6)	L5 1L			
	42 22L		Set starting values to $a_{00}$ .	
44 (7)	42 27L			
	L5 22L	By 41		
45(8)	42 14L		Set address of $a_{1,1}$ in the division	
	42 16L		and test orders.	
46 (9)	42 18L			
	L5 4F			
47 (10)	L4 2L			
	46 15L			
48 (11)	46 16L		Set addresses of $r_i$ in the division	
	46 19L		and test orders, and initial values	
49 (12)	46 21L		in the cycle.	
	46 25L			
50 (13)	L5 67L		Set both addresses in the interchange	
	42 21L		orders to 1.	
51 (14)	46 23L			
	L3(a <sub>11</sub> )F	By 8	Test and see which leading element	
52 (15)	L6(r <sub>1</sub> )F	By 10,26	$r_1$ or $a_{11}$ is larger.	
	36 18L		$=  r_1 $	
53 (16)	L5 (r <sub>1</sub> )F	By 11	$-  a_{11} $	
	66(A <sub>11</sub> )F	By 8	Divide and arrange no interchange	

LOCATION	ORDER		NOTES	PAGE 6
54 (17)	43 21L 26 20L			
55 (18)	50 66L 75(a <sub>11</sub> )F	By 9	To take care of case when a <sub>11</sub> = r <sub>1</sub> Divide and arrange	
56 (19)	66(r <sub>1</sub> )F 47 23L	By 11	interchange	
57 (20)	S1 21L 40 5F		Store ratio = -k in 5	
58 (21)	L5(r <sub>s</sub> )F 40 (1)F	By 12,31, 32 By 13,17	Copy r <sub>s</sub> and a <sub>1s</sub> into 0 and 1 <u>or</u> 1 and 0	
59 (22)	50 5F L5 284F	By 1,6,29		
60 (23)	40 (1)F 7J 1F	By 14, 19		
61 (24)	50(r <sub>n</sub> -1)F L4 F	By 12	Replace r <sub>is</sub> by r <sub>is</sub> - ka <sub>is</sub> <u>or</u> a <sub>is</sub> - kr <sub>is</sub>	
62 (25)	40(r <sub>is</sub> )F L4 2F	By 12,31 1/2	Test if result is greater than 1/2; if so record in address of 15.	
63 (26)	36 27L 47 15L			
64 (27)	L5 1F 40(a <sub>1s</sub> )F	By 7,29		
65 (28)	L5 22L L0 66L			
66 (29)	42 22L 42 27L		Increase s from i to n.	
67 (30)	L5 25L L4 67L			
68 (31)	46 25L 46 21L		Repeat until s = n+1	
69 (32)	L0 70L 36 21L			
70 (33)	L5 15L L0 61L		Test if any result > 1/2, if so divide row by 10.	
71 (34)	32 39L L5 69L		Set starting value.	

LOCATION	ORDER		NOTES
72 (35)	40 36L		
	50 39L	By 30,1/10	
73 (36)	7J(r <sub>s</sub> )F	By 35, 38	
	40(r <sub>s</sub> )F		
74 (37)	L5 36L		
	L4 67L		
75 (38)	40 36L		
	L0 68L		
76 (39)	32 35L		
	L5 4F	By 34	
77 (48)	L4 71L		Increase i
	46 4F		
78 (41)	L0 3F		Till m.
	32 7L		
79 (42)	L5 3F		Increase m.
	L4 67L		
80 (43)	40 3F		
	L0 72L		Till n + 1
81 (44)	36 2L		End of reduction
	L5 65L		$x_n = 1/10$
82 (45)	40(r <sub>n</sub> )F	By 10	Set $a_{n-1, n+1}$
	L5 22L		
83 (46)	42 50L		Set address of $x_{n-1}$ in comparison
	L5 24L		constant and substitution order.
84 (47)	46 73L		
	46 62L		
85 (48)	L5 68L		
	46 50L		
86 (49)	27 76L		
	S5 F		
87 (50)	50(r <sub>s</sub> )F	By 48,55	Evaluate scalar product $\sum_{s=1+1}^n a_{1s} x_s$
	74(a <sub>1s</sub> )F	By 46,55	using words 50 through 55
88 (51)	L4 F		
	40 F	By 76	
89 (52)	L4 2F	1/2	
	32 54L		

LOCATION	ORDER	NOTES	PAGE 8	L3
90 (53)	50 65L 7J( $r_n$ )F	1/10 by 11		If we are liable to exceed capacity reduce $x_{n-1}$ by 10.
91 (54)	26 96L L5 50L	by 35 from 52		
92 (55)	L0 67L 42 50L			Reduce s from n to i.
93 (56)	46 50L 42 58L			
94 (57)	42 60L L0 73L			Set $a_{ii}$ in comparison and division orders
95 (58)	32 49L			
96 (59)	L3( $a_{ii}$ )F L6 F 36 53L	by 57		Test if we can divide $a_{is} x_s$ by $a_{ii}$ .
97 (60)	L5 F 66( $a_{ii}$ )F	by 57		$R_1$ and $R_2 \sum_{s=i+1}^n a_{is} x_s$ Divide To
98 (61)	L6 2F S1 62L			Constant From $x_i$
99 (62)	40( $r_i$ )F L5 62L	by 47		
100 (63)	L0 75L 36 115F	by 24		Switch
101 (64)	L4 74L 26 47L			End of back substitution
102 (65)	00F 001000 0000 0000J	1/10		
103 (66)	7L 4095F LL 4095F	( $1-2^{-39}$ )		
104 (67)	00 1F 00 1F			
105 (68)	LJ( $r_n+1$ )F 40( $r_n+1$ )F	by 5 by 7		
106 (69)	7J 246F 40 246F			Test constants
107 (70)	NO( $r_n+1$ )F L4 2F	by 7		

LOCATION	ORDER		NOTES	PAGE 9
108 (71)	80 1F 00 F			
109 (72)	80 nF 00 nF	By 13 By 14		
110 (73)	50(r <sub>11</sub> )F L6 F	By 47		
111 (74)	NO 1269F L6 F	= S		
112 (75)	NO 246F L6 F			
113 (76)	00 40F 23 51L		Clear R <sub>2</sub> , then control to 51'.	
114 (77)	12 136F L5(λ <sub>n</sub> +1)F	From 39		
115 (78)	L5 94L 40 79L	From 63	Entry for zeroth solution	
116 (79)	00(L5 r <sub>0</sub> )F 00(40 x <sub>0</sub> )F	By 78, 81 From 82	Place x <sub>0</sub> <sup>(0)</sup> , ..., x <sub>0</sub> <sup>(n)</sup> in	
117 (80)	L5 79L L4 67L		λ <sub>0</sub> , ..., λ <sub>n</sub>	
118 (81)	40 79L L0 95L			
119 (82)	32 20F 26 79L			
120 (83)	L5 94L 46 86L	From 63	Entry for subsequent improved solutions	
121 (84)	46 89L 42 88L		Set initial addresses	
122 (85)	50 82F 42 89L		Waste (address used)	
123 (86)	50 (r <sub>0</sub> )F 75 (λ <sub>n</sub> )F	By 83, 91 From 93 By 16	Compute corrected x <sub>0</sub> , ..., x <sub>n</sub> and	
124 (87)	10 (10)F 66(r <sub>n</sub> )F	By 205 By 11	place in r <sub>0</sub> , ..., r <sub>n</sub> and λ <sub>0</sub> , ..., λ <sub>n</sub>	
125 (88)	S5 F L4 (λ <sub>0</sub> )F	By 84, 92		

LOCATION	ORDER		NOTES	PAGE 10
126 (89)	40(r <sub>0</sub> )F	By 84,91		
	40(λ <sub>0</sub> )F	By 85,91		
127 (90)	L5 89L			
	L4 67L			
128 (91)	40 89L			
	46 86L			
129 (92)	42 88L			
	L0 96L			
130 (93)	36 204F			
	26 86L			
131 (94)	L5 246F	r <sub>0</sub>	Starting constant	
	40 208F	λ <sub>0</sub>		
132 (95)	L5(r <sub>n</sub> +1)F	By 6		
	40(λ <sub>n</sub> +1)F	By 17	End constants	
133 (96)	40(r <sub>n</sub> )F	By 6		
	40(λ <sub>n</sub> )F	By 18		
134 (97)	L5 29F	From 54	Adjust printing parameter	
	L4 67L			
135 (98)	46 29F			
	22 45L			
	00 136K			
136 (0)	S5 F		Set link address and storage address	
	46 8L			
137 (1)	L4 4L			
	42 11L	By 25	Switch	
138 (2)	81 4F		Read in 1st sign digit, a <sub>0</sub>	
	L0 25L	-10		
139 (3)	22 10L			
	40 2F		Store a <sub>1</sub> - 10	
140 (4)	L5 F	+ M <sub>1</sub>	M <sub>1</sub> = N <sub>1</sub> - D <sub>1</sub> /2	
	66 1F	+D <sub>1</sub> /2	M <sub>1</sub> /D <sub>1</sub> +2 = N <sub>1</sub> /D <sub>1</sub> + 2 - 1	
141 (5)	10 1F		N <sub>1</sub> /D <sub>1</sub> - 1/2	
	SJ 42L			
142 (6)	40 F			
	L1 F	From 15		

LOCATION	ORDER	NOTES		PAGE 11
143 (7)	40 1F	By 10'	Choose and store either $+ N_1/D_1$ or	L3
144 (8)	L5 (2)F	By 0'	$-N_1/D_1$	
	L5 8L			
145 (9)	L4 4L		Increase storage address	
	46 8L			
146 (10)	L5 2F	From 3		
	42 7L			
147 (11)	L0 23L	-2		
	32(q+1)F	By 1' and 26F	= N return to main routine or to residue computer (switch)	
148 (12)	L5 24L	+ 5		
	40 1F		$D_0/2 = 5$	
149 (13)	41 F			
	81 4F		Read in digit $a_1$	
150 (14)	L0 25L	-10		
	40 2F		Store $a_1 - 10$	
151 (15)	32 6L		Check for sign or terminating symbol	
	L4 24L	+5		
152 (16)	40 F	From 22'	Store $a_1 - 5 = M_1, M_1$	
	81 4F		Read in $a_1$	
153 (17)	50 1F		$D_1/2$ in $R_2$	
	40 2F		Store $a_1$	
154 (18)	L0 25L	-10	$a_1 = 10$	
	32 3L		Check for sign or terminating symbol	
155 (19)	75 25L	x 10	$D_1/2 \times 10$ in $R_2$	
	85 26L			
156 (20)	40 1F		Store $D_{i+1}/2 = D_1/2 \times 10$	
	50 25L			
157 (21)	75 F	x $M_1$	$10 M_1 2^{-78}$	
	00 39F		$10 M_1 2^{-39}$	
158 (22)	L4 2F	+ $2_{i+1}$	$M_{i+1} = 10 M_1 + a_{i+1}$	
	26 16L			
159 (23)	00 F			
	00 2F			
160 (24)	00 F			
	00 5F			



LOCATION	ORDER	NOTES
161 (25)	00 F 00 10F	
162 (26)	00 F L5 203F	Set initial addresses
163 (27)	40 29L 41 F	Clear $R_1, R_2, 0$
164 (28)	50 F S5 F From 33	
165 (29)	00(50 $\lambda_0$ )F By 26 00(74 $r_0$ )F By 32	Accumulate scalar product in 0
166 (30)	L4 F 40 F	
167 (31)	L5 29L L4 104F	Advance addresses and test for end of scalar product
168 (32)	40 29L L5 43L	
169 (33)	L0 29L 32 28L	
170 (34)	L5 F 66 ( $\lambda_n$ )F By 16	Compute $\delta_1$ and place in 0
171 (35)	S5 F 40 F	
172 (36)	19 (10)F By 205 L2 F	$2^{-11} -  \delta_1 $
173 (37)	32 40L L5 36L	If $ \delta_1  > 2^{-11}$ reduce shift orders by 2 and stop to accept matrix tape again.
174 (38)	L0 182F 46 36L	
175 (39)	46 41L 46 124F	
176 (40)	24 37F L5 F	Put $2^{10} \delta_1$ in $r_n$ and leave
177 (41)	00 (10)F By 205 50 F	Waste
178 (42)	40( $r_n$ )F By 9 22(q+1)F By 2	

LOCATION	ORDER		NOTES	PAGE 13	L3
179 (43)	50( $\lambda_n$ )F	by 15			
	74( $r_n$ )F	by 10	End test		
	00 180K				
180 (0)	40 F		Store x at 0		
	15 22L				
181 (1)	S4 F	by 1'			
	46 1L				
182 (2)	40 2F				
	92 961F		Space		
183 (3)	L5 F				
	36 5L				
184 (4)	92 706F		- Sign		
	22 5L				
185 (5)	92 644F		+ sing		
	50 161F		$10 \times 2^{-39}$		
186 (6)	22 7L				Construct $10^p \times 2^{-39}$
	46 1L				and place in 1.
187 (7)	75 161F		$10 \times 2^{-39}$		
	L5 1L				
188 (8)	L0 15L				
	32 6L				
189 (9)	S5 F				
	40 1F				
190 (10)	50 L		From $ x  + 1/2 \times 10^{-p}$		
	67 1F				
191 (11)	L7 F				
	S4 120F				
192 (12)	40 F				
	50 F				
193 (13)	75 161F				
	00 36F		Print one digit		
194 (14)	82 4F				
	10 40F				
195 (15)	S4 10F				
	40 F				

LOCATION	ORDER		NOTES	PAGE 14
196 (16)	L5 1L			
	L0 12L		Should we space?	
197 (17)	46 1L			
	00 9F			
198 (18)	36 19L			
	92 643F			
199(19)	L5 2F			
	L0 15L			
200 (20)	46 2F			
	00 10F		Have we printed the last digit?	
201 (21)	32 12L			
	22 30F		Return to program	
202 (22)	02 1013F			
	00 1F		Constant	
203 (23)	50 208F	$\lambda_0$	A starting constant	
	74 246F	$r_0$		
204 (24)	L5 172F	From 130	Increase shift orders by 4	
	L4 37F			
205 (25)	46 172F			
	46 177F			
206 (26)	46 124F			
	22 26F			
207 (27)	00 245F		$r_0 - 1$	
	00 245F		$r_0 - 1$	
	24 3N			