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Library Routine M 24 - 260

TITLE: Complete Linear Matrix Equation Solver and General
Matrix Inversion Routine Using Drum Storage. (DOI Only)
TYPE: Complete Program (DOI built - in)
DESCRIPTION: This routine solves the matrix equation $A X = B$ given
A and B where

1. A is a non-singular square matrix of order n
2. B is of size (n x m).

The magnitudes of n and m are governed by

$$n + m \leq 164$$
$$n^2 + 2nm + 3m \leq 20,420.$$

In case of inversion ($B = I$) $n = m$ and the limit is
 $n \leq 82$.

There are two options on input and two options on
output:

- Input: 1. From tape
2. From drum

- Output: 1. Tape output
2. Results stored on drum.

This routine uses the same method of solution as M 13.
Characteristic times for calculation (input and output
not included) are:

Inversion: n = 20 - ~ 90 sec.
 n = 40 - ~ 600 sec.
Linear equation: n = 10, m = 1 - ~ 8 sec.
 n = 10, m = 4 - ~ 15 sec.

For higher orders, the time increases approximately
as n^3 .

Note: Due to the large amount of drum recording necessary
in this routine, it is not advisable to use the routine
for low order matrices, which could be accommodated by
M 13 or M 14 which are much faster.

TAPE PREPARATION:

Each matrix equation to be solved uses some tape input: either a data tape containing the matrix or a parameter tape, depending on the option. In all cases scaling is the same as in M 13.

Input Option 1: When matrix A and/or B are to be read from tape, they appear in the form

$\pm a_{11}$	$\pm a_{12}$	$\pm a_{13}$	\dots	$\pm a_{1n}$	C
	$\pm b_{11}$	$\pm b_{12}$	\dots	$\pm b_{1m}$	C
$\pm a_{21}$	$\pm a_{22}$	\dots	\dots	$\pm a_{2n}$	C
	$\pm b_{21}$	$\pm b_{22}$	\dots	$\pm b_{2m}$	C
\dots					
$\pm a_{n1}$	$\pm a_{n2}$	\dots	\dots	$\pm a_{nn}$	C
	$\pm b_{n1}$	$\pm b_{n2}$	\dots	$\pm b_{nm}$	C P

where $\pm a_{ij}$ and $\pm b_{ij}$ are the elements of the matrices A and B. C is a control character for calculations. P is a control character for output.

If the problem is the solving of a linear matrix equation (as opposed to inversion) then the C after each row is to be the sexadecimal character N. If the problem is the inversion of A, C is to be a J and the elements of B are not punched. B = I will automatically be constructed by the program when C = J. See OUTPUT for description of the P digit.

With this option, drum locations 2560 to $(2559 + n(m + 1) + \frac{n(n + 1)}{2})$ will be used by the program.

Input Option 2: The matrices A and/or B can be stored on the magnetic drum (as from previous programs) prior to reading in this routine. The matrices must be stored by row in same order as they would be punched on tape with a sum check after each row of each matrix. (This routine uses Y 1 for drum transfers.) In the case of inversion B need not be stored with A on the drum.

The input tape has form

C + n + m + d N P

where

C: Control digit where F: Solves linear equation ($B \neq I$)

L: Inversion ($B = I$)

n: Order of matrix A

m: Columns in matrix B

d: Drum location of first element of A

N: Sexadecimal character N

P: Print control character (See OUTPUT).

Drum usage with this option is as follows:

1. If matrix A is to be inverted ($C = L$), it should be stored above the drum location given by

$$2561 + \frac{n(n+1)}{2}$$

to allow room for forming $B = I$ on the drum.

2. The upper limit on drum storage is 12769.

3. Working storage on the drum is from 2560 to

$$(2559 + n(m+1) + \frac{n(n+1)}{2}).$$

OUTPUT:

The digit P determines the form of output. If

$$1 \leq P \leq S$$

output will be on tape by column where P determines number of digits to be punched for each element. Each column will be followed by a scaling factor and an N. (See M 13 for complete description of this mode).

If $P = L$ the results of the computation will be placed consecutively on the drum by row, starting in 2560, a sum check following each row. After the last row the scaling constants for each column will be written as an extra row of the matrix.

NOTE:

1. Upon completion of one problem, the program stops on a 2406K.

A black switch start will start a new problem.

2. A stop with

FF 010

42 ---

signifies a sum check stop in Y 1.

3. A stop with

-- --
FF 011

signifies a sum check error in reading in the program. Another attempt should be made to read it in.

LOCATION	ORDER	NOTES	PAGE 1	M 24
	Library Routine X 1 - 218 00 9K		Decimal Order Input	
	Library Routine N 12 - 225 00 48K		Infraput	
	Library Routine Y 1 - 199 00 88K		Transfer Block of Words from the Memory to the Drum or from the Drum to the Memory	
	Library Routine P 2 - 52 00 106K		Print (A) with or without Sign to n Places as Determined by a Program Parameter	
0	41 6F	}	Clear Counters	
	41 7F			
1	50 368F	}	Call in first row	
	50 1L			
2	26 9F	}	Use Drum?	
	10 250L			
3	36 20L			
	40 8F	}	Store I and S 1 to 0	
4	L5 256L			
	40 32L			
5	22 37L			
	K5 61L	}	Set y addresses	
6	42 12L			
	L5 30F			
7	46 44L			
	46 49L			
8	46 151L			
	46 167L			
9	46 207L			
	10 20F			
10	42 53L			
	42 151L			
11	42 173L			
	42 257L			

LOCATION	ORDER		NOTES
12	42 261L 22 F	6	
13	K5 367F 42 19L		
14	L5 30F 46 76L		
15	46 117L 46 258L		
16	10 20F 42 80L		
17	42 83L 42 86L		
18	42 91L 42 97L		
19	42 259L 22 F	13'	
20	L0 250L 40 8F	(3)	
21	52 368F 50 21L		
22	26 9F L5 368F		
23	42 3F 00 20F		
24	46 33L L5 369F		
25	42 4F 00 20F		
26	46 46L L5 370F		
27	L4 256L 40 32L		
28	L5 256L L0 32L		
29	36 36L L5 13L		

Set t addresses

n
Set Drum parameters
and S 1 to d

m

d

S 1 = 0?

LOCATION	ORDER		NOTES	PAGE 3	
30	L4 33L 46 30F		Set y in (M12)		
31	50 368F 50 31L				
32	26 48F 00 F	4' 47' 27' 34' 24	Read row r of A from Drum.		
33	00 F F5 32L				
34	L4 3F 40 32L				
35	40 45L 22 40L		Read row r of A from tape.		
36	50 368F 50 36L				
37	26 9F L5 30F		Set n, m = n		
38	L0 36L L0 20F				
39	42 3F 42 4F		Set y addresses		
40	22 40L 50 40L				
41	22 5L F5 8F		B = I?		
42	32 53L L5 256L		S 1 = 0?		
43	L0 32L 36 49L				
44	50 F 50 44L	7	Read row # of B from Drum.		
45	26 48F 00 F	35 48			
46	00 F F5 45L		26		
47	L4 4F 40 32L				

LOCATION	ORDER		NOTES	PAGE 4
48	40 45L 26 59L		Read row r of B from tape.	
49	50 F 50 49L	7'		
50	26 9F 15 30F		Set n	
51	10 49L 10 20F			
52	42 4F 50 5L		Set t addresses (to 62L)	
53	26 13L 41 F	10 54'		
54	75 53L 42 53L		Clear y to $y + n - 1$	Set B = I
55	10 5F 10 257L			
56	32 53L 15 257L		Augment A with I 1/10	
57	1A 6F 42 58L			
58	15 254L 40 F	57'	Set t in N 12	
59	15 4F 50 250L			
60	00 20F 1A 44L		n + m	
61	46 30F 22 52L			
62	22 62L 15 3F		Preset Drum commands	
63	1A 4F 26 64L			
64	00 20F 46 78L			
65	46 119L 15 255L			

LOCATION	ORDER		NOTES	PAGE 6	M 24	
84	26 88L					
	50 F	68	}			
85	S3 696F			x = 0?		
	32 87L		}			
86	50 253L					
	75 F	17'	}			
87	66 F	72'		comp $\left \frac{t}{x} \right $		
	47 92L					
88	41 5F					
	SL 532F					
	40 2F					
89	41 F					
	22 90L					
90	L5 F	68' 101	}			
	40 F	79' 82				
91	L5 F	18 90'		Interchange		
92	40 F	80 87'				
	50 2F		}			
93	7J 1F			$x' = \begin{matrix} t - kx \\ x - kt \end{matrix}$		
	L4 F		}		Eliminate between rows i and r	
94	40 F	73				
	L3 F	69				
95	L6 5F			Store $ x' _{mx}$		
	36 97L		}			
96	L7 F	73' 101'				
	40 5F		}			
97	L5 1F			Complete		
	40 F	18' 98'	change			
98	F5 97L		}			
	42 97L					
99	42 91L					
	L5 94L					
100	L4 251L					
	40 94L					
101	42 90L					
	46 96L					

LOCATION	ORDER		NOTES	PAGE 7	M 24
102	L0 258L		} More elements } Singular? $ a_{r_j} _{mx} = 0$ } Determine amount to scale row i } Scale row i		
	32 90L				
103	L3 5F				
	36 248L				
104	L5 252L				
	40 112L				
105	LL 5F				
	32 108L				
106	L5 89L				
	46 112L				
107	26 111L				
	F5 112L				
108	42 112L				
	L5 5F				
109	00 1F				
	40 5F				
110	LL 5F				
	32 107L				
111	50 250L				
	L5 F	69' 115			
112	10 F	106' } 104'			
	00 F	108' }			
113	50 250L				
	40 F	70 114'			
114	F5 113L		} Scale to $\frac{1}{2} > a_{r_i} _{mx} \geq \frac{1}{4}$		
	42 113L				
115	42 111L				
	L0 259L				
116	32 111L				
	26 117L				
117	J0 F	15 135'			
	50 117L				
118	26 48F	66' }			
	00 F	121' }			
119	00 F	65 123	} Put row i back on drum		
	L5 119L				

LOCATION	ORDER	NOTES	PAGE 8
120	10 20F F4 118L		
121	40 118L 40 77L		
122	L5 119L L0 251L		
123	46 119L 46 78L		
124	L5 49L 46 30F		
125	22 125L 50 125L		Set y addresses
126	22 5L L5 258L		
127	46 30F 50 127L		Set t addresses
128	26 13L F5 7F		
129	42 7F 26 67L		$i \rightarrow i + 1$
130	L3 7F L6 6F		$i = r?$
131	36 135L F5 6F		
132	42 6F L0 3F		$r \rightarrow r + 1$
133	32 145L 41 7F		$r = n?$
134	26 28L 00 F		Clear i and read in next row
135	L5 138L 46 117L		
136	L3 6F 32 137L		
137	26 117L 41 5F		

LOCATION	ORDER		NOTES	PAGE 9
138	L3 F	74 142		
	L6 5F			
139	32 140L			
140	L7 F	70' 141		Put row r on Drum
141	40 5F			
142	F5 139L			
143	42 139L		i → i + 1	
144	00 20F			
145	46 138L			
146	F5 7F			
147	42 7F			
148	10 3F			
149	10 4F		i = n?	
150	36 103L			
151	26 138L			
152	81 4F			
153	00 20F			
154	46 210L		Set punch and S2	
155	14 251L			
156	00 15F			
157	32 149L			
158	49 8F			
159	26 150L			
160	41 8F			
161	41 5F		Clear k	
162	L5 254L		Fetch $S_0 = 1/10$	
163	40 F	8	Store S	
164	L3 F	10'		
165	36 248L		Sing? ($S < 2^{-39}$)	
166	L5 118L			
167	40 162L		Set initial Drum addresses	
168	L5 119L			
169	46 163L			
170	41 6F		Clear counters	
171	41 7F			
172	L5 169L			

LOCATION	ORDER		NOTES	PAGE 10 M 24
156	42 209L		Adjust drum addresses	
	L5 162L			
157	F0 4F			
	F0 6F			
158	40 162L			
	40 193L			
159	L5 163L			
	L4 251L			
160	46 163L			
	46 194L			
161	50 532F		Call in row (n - r)	
	50 161L			
162	26 48F	153		
	00 F	158		
163	00 F	154 160		
	L5 88L		Reset addresses	
164	F4 6F			
	42 170L			
165	L4 5F			
	42 167L			
166	L5 167L			
	46 170L			
167	50 F	8'	Compute $\sum_{r=1}^n a_{ir} x_r$	
	71 F	165'		
168	40 F			
	22 174L			
169	00 327F			
	S5 368F			
170	50 F	166'	175	
	74 F	164'		
171	L4 F		$ \Sigma < \frac{1}{2} ?$	
	40 F			
172	LL F			
	32 174L			
173	50 254L		If not, rescale	
	7J F	11		

LOCATION	ORDER		NOTES	PAGE 11 M 24	
174	26 151L L5 170L		Advance addresses (n - r + 1) times for row r		
175	L0 251L 40 170L				
176	42 180L 42 183L				
177	46 185L F5 7F				
178	40 7F L5 6F				
179	L0 7F 32 169L				
180	41 7F L3 F	176		Reset i	
181	36 248L L6 F			Sing? $a_{ii} = 0$	
182	36 173L 26 183L			Division bad?	
183	L5 F 66 F	176'		$x_r = \frac{\sum}{a_{rr}}$	
184	22 184L S1 532F				
185	40 F L3 8F	177	S2 = 0?		
186	32 194L L3 5F		k = 0?		
187	32 194L L5 185L				
188	L4 169L 46 191L				
189	L5 184L L4 5F				
190	L4 6F 42 191L		Restore row (n - r) of augmented matrix to drum with $x_{r,k-1}$		
191	L5 F 40 F	188' 190'			

LOCATION	ORDER		NOTES	
192	JO 532F 50 192L			
193	26 48F 00 F	158'	Count n rows	
194	00 F F5 6F	160'		
195	42 6F L0 3F			
196	36 197L 22 156L			
197	L3 8F 36 209L			S2 = 0?
198	L5 260L 40 200L			No
199	41 F 26 200L			
200	L5 F 40 F	198' 202		
201	L5 200L L4 251L			
202	40 200L F5 F			Store column k in WM (700 to 700+n) and save scaling factor, S _k .
203	42 F L0 4F			
204	36 205L 26 200L			
205	L5 206L L4 5F			
206	42 207L 50 860F			
207	L5 F 40 F	9 206		
208	26 214L 00 367F			
209	92 131F L5 F	156 212	Yes	

LOCATION	ORDER		NOTES	PAGE 13
210	50 F	146'		
	50 210L			
211	26 88F		Punch column k	
	F5 209L			
212	42 209L			
	F0 261L			
213	36 209L			
	92 770F			
214	F5 5F			
	42 5F			
215	10 4F		k → k + 1 k ≠ m?	
	32 216L			
216	22 150L			
	L3 8F		S2 = 0	
217	32 248L			
	L5 255L			
218	40 228L			
	40 235L			
219	L5 4F			
	00 20F			
220	46 236L			
	46 247L			
221	L5 4F			
	L4 3F		Set initial addresses for Drum transfer	
222	00 20F			
	46 229L			
223	10 20F			
	L4 208L			
224	42 235L			
	L5 207L			
225	46 234L			
	L5 85L			
226	46 233L			
	41 2F			
227	50 368L			
	50 227L			

LOCATION	ORDER		NOTES	PAGE 14
228	26 48F 00 F	218 231		
229	00 F L5 229L	222' 232		
230	10 20F F4 228L			
231	40 228L L5 229L			
232	L0 251L 46 229L			
233	L5 F 40 F	226 241 224 242		
234	J0 F 50 234L	225 237'		
235	26 48F 00 F	218' 239	Isolate X on Drum by row	
236	00 F L5 234L	220		
237	L0 251L 46 234L			
238	L5 235L F4 4F			
239	40 235L 40 246L			
240	L5 233L L4 251L			
241	46 233L L0 250L			
242	42 233L F5 2F			
243	42 2F L0 4F			
244	36 245L 26 227L			

LOCATION	ORDER		NOTES	PAGE 15	M 24
245	JO 860F		}		
	50 245L				
246	26 48F	239'	}	Store scale	
	00 F				
247	00 F	220'	}	constants on	drum
	22 248L				
248	92 898F		}	F	LF and CR (3)
	92 139F				
249	24 L		}	REPEAT	
	00 F				
250	00 F		}		
	00 2F				
251	00 1F		}		
	00 1F				
252	10 1F		}		
	00 1F				
253	7L 4095F		}		
	LL 4095F				
254	00 F		}		
	00 1000 0000 0000J				
255	26 48F		}	Constants	
	00 2560F				
256	26 48F		}		
	00 F				
257	K6 13L		}		
	41 F	11'			
258	NO F	15'	}		
	L3 F				
259	JO 250L		}		
	40 F	19			
260	L5 368F		}		
	40 696F				
261	12 131F		}		
	L5 F	12			

LOCATION	ORDER	NOTES	PAGE 16	M 24
262	41 F	Sum check routine		
	L5 3F			
263	L6 F			
	40 F			
263	F5 368F			
	42 368F			
264	L0 375F			
	32 368F			
265	L3 F			
	36 249L			
266	82 40F			
	FF 17F			
267	22 373F			
	00 F			
268	N1 F			
	L5 376F			
269	N3 1746F	Σ check constant		
	50 3893F			
	26 368N			

RT: 11/3/60

DATE	<u>April 1, 1959</u>
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