TITLE: Symmetric Matrix Inversion

TYPE: Entire program

CAPACITY: $2 \leq n \leq 139$ where n is the order of the matrix

ACCURACY: A function of the order and conditioning of the matrix

DESCRIPTION: This routine inputs a symmetric matrix as a lower triangular
                matrix, and inverts it. Input may be from tape or from the
                drum. Output may be stored on the drum as a lower triangular
                matrix, or punched to "d" decimal places $(1 \leq d \leq 12)$ as a
                square matrix, a lower triangular matrix, or a column of the
                diagonal entries.

METHOD OF USE: STOPS

                Master tape   3022K

                Parameter tape   24068

                Data tape   2022K

                At the end of a problem the program will stop on a 2022K order.
                Raising the black switch will begin a new problem with new
                parameters. Moving the white switch up and down will begin
                a new problem with the old parameters.

THE PARAMETER AND DATA TAPES:

                The parameter tape is punched as + n + s + d + pT

                i) n is the order of the matrix

                ii) s is some integer power of 10. This is used to scale the
                input matrix so that the largest element in the matrix is as
                close to 1/2 in machine representation as possible, without
                equalling or exceeding it.

                iii) d is the number of decimal places in the output

                iv) p is the input option

                        a) 0 designates tape input. It need not be punched.

                        b) any non-zero number designates drum input beginning
                           at location 2906.

v) T is the output option

    a) N designates punching a lower triangular matrix

    b) J designates punching a square matrix

    c) F designates punching a column of the diagonal matrix

    d) L designates storing of the lower triangular matrix
       on the drum beginning at drum location 2906, with a
       sum check at the end of each row.

The data tape is punched as signed fractions by rows. It is
in the form of a lower triangular matrix and the matrix is
terminated by a J.

EXAMPLE 1:   Input the following matrix from tape and punch the inverted
matrix as a square matrix to 5 decimal places

$$\begin{vmatrix} .5 & -.6 & -.6 \\ -.6 & .8 & .9 \\ -.6 & .9 & -.9 \end{vmatrix}$$

The largest element would be greater than 1/2 in machine re-
presentation. Scale the matrix by 10^{-1}, that is s = -1. The
parameter tape is punched as +3 -1 +5 +J. The data tape is punched
as +5 -6 +8 -6 +9 -9J.

EXAMPLE 2:   The matrix in example 1 is already stored on the drum as a
lower triangular matrix, beginning at location 2906. Invert
and punch to 6 decimal places as a column of diagonal entries.
The parameter tape is punched as +3 -1 +6 +1F.

EXAMPLE 3:   Invert the following matrix and store on the drum.

$$\begin{vmatrix} .1 & 0 & 0 & 0 \\ 0 & .2 & 0 & 0 \\ 0 & 0 & .3 & 0 \\ 0 & 0 & 0 & .4 \end{vmatrix}$$

The largest element is less than 1/2 in machine representation.
Thus s = 0. Notes also that d = 0 since there is no tape out-
put. The parameter tape is punched as +4L. The data tape is
punched as +1 + +2 ++ +3 +++ +4J.

THE OUTPUT MATRIX:

The elements of the punched output matrix are punched as signed fractions. There are four modes of output.

i) <u>Square matrix</u>. The elements are punched by rows with an N after each row and a J after the matrix.

ii) <u>Lower triangular matrix</u>. The elements are punched by rows and 2 line feeds and carriage returns follow each row. A J will follow the matrix.

iii) <u>Column of diagonal entries</u>. Each element is followed by a line feed and carriage return.

iv) The output is stored as a lower triangular matrix on the drum beginning at location 2906. A sum check is stored after each row.

The punched outputs have a d-place number appearing after the output matrix. This indicates the position of the decimal point as lying after the column in which a 1 appears.

EXAMPLE 4:

The output to example 1 appears as

+18889 +13333 +00741N
+13333 +10000 +01111N
+00741 +01111 -00494NJ
+01000

This would be interpreted as

$$\begin{vmatrix} 18.889 & 13.333 & 0.741 \\ 13.333 & 10.000 & 1.111 \\ 0.741 & 1.111 & -0.494 \end{vmatrix}$$

In general: Let A be the matrix (of fractions) punched on the input tape and let B be the output matrix (with decimal points in positions as specified above).

Then:

$$AB = I(10^{-s-1})$$

where I is the unit matrix and s is the input scale factor discussed above. Hence

$$A^{-1} = 10^{s+1}B$$

DURATION: It takes a minute to read in the master tape. The computation time is given by $[0.004n^3 + 0.08n^2 + 0.1n]$ seconds.

PROGRAMMED STOPS:

FF000 Matrix is singular

FF001 Matrix is overscaled

FF002 Diagonal element too small

FF003 Master tape read incorrectly. Reread.

Moving the white switch up and down will start a new problem at stops FF000 to FF002.

MATHEMATICAL METHOD:

Let A be the matrix to be inverted. Let L be a lower, U an upper triangular matrix, and D a diagonal matrix. Then A = LDU. If A is symmetric $A = (LDU)^T = U^T D^T L^T$. That is $L = U^T$, $D = D^T$ and $U = L^T$

$$A^{-1} = (LDU)^{-1} = U^{-1} D^{-1} L^{-1} = (L^T)^{-1} D^{-1} L^{-1}. \qquad (*)$$

The program stores A as a lower triangular matrix on the drum. By a series of row transformation on A, L, and D are obtained. L^{-1} is obtained by the following:

$$\ell_{jk}^{-1} = \begin{cases} 0 \text{ if } j < k \\ 1 \text{ if } j = k \\ -\sum_{m=j+1}^{k} \ell_{jm}^{-1} \ell_{mk} \text{ if } j > k \end{cases}$$

D^{-1} is obtained by the following:

$$d_{jk}^{-1} = \begin{cases} 0 \text{ if } j \neq k \\ \dfrac{1}{d_{jj}} \text{ if } j = k \end{cases}$$

A^{-1} is obtained from equation (*).

A necessary condition for singularity is that $d_{jj} = 0$. In machine representation, due to the accumulation of round-off

errors, it is difficult to detect a near singular matrix. The
results should always be checked when singularity is detected
by the program.

DATE   March 20, 1961

PROGRAMMED BY   F. Shimamoto

APPROVED BY

nj

LOCATION	ORDER	NOTES
	003K	
0	00 F	
	00 607F	Location of vectors A, B, C
1	00 F	
	00 746F	
2	00 F	
	00 885F	
3	00 F	
	00 2906F	Location of matrix on drum
	0010K	
10 (N)	00 F	
	00 F	Constants
11 (S)	00 F	
	00 F	
12 (P)	00 F	
	00 F	
13 (I)	00 F	
	00 F	
14 (O)	00 F	
	00 F	
15 (MS)	00 F	
	00 F	
16 (Z)	00 F	
	00 F	
17 (V1)	7J S	
	34 0S3	
18 (V2)	00 F	
	00 S4	
19 (V3)	L4 S5	
	40 S5	
20 (1)	00 F	
	00 1F	
21 (1-1)	00 1F	
	00 1F	

LOCATION	ORDER	NOTES
22	(+) 00 F	
	00 10F	
23	(-) 00 F	
	00 1000 0000 0000 J	
24	(+1) 00 F	
	00 9999 9999 9999 J	
25	(34) 00 F	
	00 34F	
26	(C1) 00 F	
	00 F	
27	00 F	
	00 2F	
28	(C2) 26 (Y1)	
	00 2S6	
	00545K	
545	(MTR) JO 27F	
	50 L	Store part 1 of program on drum
546	26 (Y1)	
	00 2560F	
547	00 116F	
	26 999F	
548	JO 29F	
	50 3L	
549	26 (Y1)	
	00 2677F	Store part 2
550	00 125F	
	26 999F	
551	JO 52F	
	50 6L	
552	26 (Y1)	
	00 2803F	Store part 3
553	00 102F	
	26 999F	
554	50 27F	
	50 9L	

LOCATION	ORDER	NOTES
555	26 (Y1)	
	00 2560F	Playback part 1
556	00 116F	
	22 29F	
557	50 29F	
	50 12L	Playback part 2
558	26 (Y1)	
	00 2677F	
559	00 125F	
	26 29F	
560	50 52F	
	50 15L	Playback part 3
561	26 (Y1)	
	00 2803F	
562	00 102F	
	26 53F	
563	20 9L	
	22 19L	End of program: bl. sw. to read new parameter, wh. sw. to retain old parameter
564	50 27F	
	50 19L	
565	26 (Y1)	
	00 2560F	Playback part 1
566	00 116F	
	26 31F	
567	(Y1), 00K	Drum transfer routine (Y1)
	00 29K	
29	(IN) 52 10F	
	50 L	Read in parameters
30	26 (N12)	
	40 14F	
31	L3 (S)	
	36 14L	
32	L5 (S)	
	32 8L	
33	40 4F	
	L5 (▽)	

LOCATION	ORDER	NOTES PAGE 4 M 34
34	40 3F	
	F5 4F	Store negative scaler
35	36 14L	
	40 4F	
36	50 3F	
	7J (÷)	
37	26 5L	
	L1 (S)	
38	40 4F	
	L5 (+)	
39	40 3F	
	F5 4F	Store positive scaler
40	36 14L	
	40 4F	
41	50 3F	
	75 (+)	
42	S5 F	
	26 10L	
43	L5 (1)	
	40 5F	
44	L1 (N)	
	40 4F	
45	L3 (II)	
	32 46L	Input option test: jump to tape input
46	L3 (S)	
	32 64L	
47	50 S3	
	50 18L	Playback row of matrix from drum
48	26 (Y1)	
	00 S6	
49	00 1F	
	50 20L	
50	26 (M)	
	22 22L	
51	JO S3	
	50 22L	

LOCATION	ORDER	NOTES
52	26 (Y1)	Store row of matrix on drum
	00 S6	
53	00 1F	
	F5 4F	
54	32 64L	
	40 4F	
55	F5 5F	
	40 5F	
56	L4 19L	
	40 19L	
57	40 23L	
	L5 20L	
58	L4 (1-1)	
	46 20L	
59	46 24L	
	22 18L	
60 (M)	K5 F	Scale a row of matrix by $10^{\pm s}$ routine
	42 38L	
61	42 43L	
	L1 5F	
62	40 6F	
	L1 (S)	
63	32 40L	
	L5 (V1)	
64	42 36L	
	42 37L	
65	50 3F	
	75 F	
66	00 39F	
	40 F	
67	F5 6F	
	32 F	
68	40 6F	
	F5 36L	
69	26 35L	
	L5 (V1)	

LOCATION	ORDER	NOTES
70	40 42L	
	50 3F	
71	7J F	
	40 F	
72	F5 6F	
	32 F	
73	40 6F	
	L5 42L	
74	L4 (1-1)	
	26 41L	
75	50 F	
	41 6F	
76	L5 74L	
	40 23(N12)	
77	50 S3	
	50 48L	Read in data
78	24 (N12)	
	F5 6F	
79	40 6F	
	L0 5F	
80	32 52L	
	L5 2F	
81	26 4(N12)	
	L3 (S)	
82	32 55L	
	50 53L	
83	26 (M)	
	22 55L	
84	J0 S3	
	50 55L	
85	26 (Y1)	
	00 S6	Store a row of A on drum
86	00 1F	
	F5 4F	
87	32 64L	
	40 4F	

LOCATION	ORDER	NOTES
88	F5 5F	
	40 5F	
89	L4 56L	
	40 56L	
90	L5 57L	
	L4 (1-1)	
91	46 57L	
	L5 (V1)	
92	46 21(N12)	
	41 6F	
93	22 51L	
	L5 (N)	
94	L4 66L	
	40 5F	
95	50 (1)	
	41 S5	Clear C vector
96	F5 66L	
	42 66L	
97	L0 5F	
	32 69L	
98	22 66L	
	L5 (N)	
99	00 20F	
	L4 (N)	
100	40 193F	
	L5 (1)	
101	40 256F	
	40 2F	
102	41 (MS)	
	22 12(MTR)	
103	22 49(IN)	
	00 F	
104	(N12)00K	Input routine (N12)
	00K	
0	L3 F	
	40 607F	

LOCATION	ORDER	NOTES
1	22 545F	
	50 F	
	26 L	
	26 1N	
	0029K	
29	F5 (V1)	
	42 257F	
30	F5 (1)	
	40 3F	
31	L1 (N)	
	L4 (1)	
32	40 (C1)	
	40 7F	
33	L5 (1)	
	40 6F	
34	85 11F	
	00 S6	
35	40 S4	
	L5 S5	
36	40 4F	
	22.8L	
37	50 S3	
	50 8L	Playback one row of matrix
38	26 (Y1)	
	00 2S6	
39	00 2F	
	L1 1S5	
40	L4 4F	
	40 1F	
41	L5 S3	
	40 1S3	
42	L3 1F	
	32 25L	
43	L7 1F	
	00 20F	

LOCATION	ORDER
44	46 20L
	46 24L
45	L1 1F
	32 23L
46	L4 4F
	40 4F
47	L5 (V2)
	42 19L
48	42 20L
	L5 F
49	10 F
	40 F
50	F5 20L
	L0 24L
51	36 25L
	F5 19L
52	22 18L
	L5 1S4
53	10 F
	40 1S4
54	41 1F
	L5 S3
55	40 F
	L7 S4
56	L2 F
	32 30L
57	F5 1F
	40 1F
58	L5 F
	10 1F
59	26 26L
	L3 1F
60	32 39L
	L7 1F
61	00 20F
	46 36L

Test: Is $a_{jj} - a_{jk} \leq 0$?

LOCATION	ORDER	NOTES
62	F1 2F	
	40 5F	
63	L5 (V1)	
	42 35L	
64	42 36L	
	L5 F	
65	10 F	
	40 F	
66	F5 5F	
	40 5F	
67	32 39L	
	F5 35L	
68	22 34L	
	L7 4F	
69	L4 (1)	
	00 20F	
70	46 45L	
	L3 S4	
71	36 124L	
	50 S3	
72	75 (+1)	
	66 S4	Form $\dfrac{a_{jk}}{a_{jj}}$
73	S5 F	
	40 8F	
74	10 1F	
	00 1F	
75	32 46L	
	40 S3	
76	L1 6F	
	40 5F	
77	F5 43L	
	42 50L	
78	F5 42L	
	42 51L	
79	42 54L	
	L5 F	

LOCATION	ORDER	NOTES
80	40 F	
	L5 F	
81	40 9F	
	36 53L	
82	50 8F	} Form ℓ_{jk}
	79 F	
83	L4 9F	
	40 F	
84	F5 5F	
	40 5F	
85	32 58L	
	F5 50L	
86	42 50L	
	F5 51L	
87	22 49L	
	F1 2F	
88	40 F	
	L5 (V1)	
89	42 61L	
	42 62L	
90	L7 5F	
	L2 F	
91	32 63L	
	L7 F	
92	40 5F	
	F5 F	
93	40 F	
	36 66L	
94	F5 61L	
	26 60L	
95	F1 2F	
	40 9F	
96	LL 5F	
	32 111L	
97	F5 F	
	42 71L	

LOCATION	ORDER	NOTES PAGE 12 M 34
98	F0 (1)	
	L0 1F	
99	40 F	
	L3 F	
100	32 78L	
	L5 (V1)	
101	42 73L	
	42 75L	
102	50 (1)	
	L5 F	
103	10 2F	
	00 1F	
104	32 75L	
	40 F	
105	F5 9F	
	32 78L	
106	40 9F	
	F5 73L	
107	26 72L	
	L5 1S5	
108	L4 F	
	40 1S5	
109	J0 S3	
	50 80L	Record a row of L on drum
110	26 (Y1)	
	00 2S6	
111	00 2F	
	F5 7F	
112	32 93L	
	40 7F	
113	L5 10L	
	L4 (1-1)	
114	40 10L	
	46 82L	
115	42 78L	
	42 79L	

LOCATION	ORDER
116	F5 2F
	40 2F
117	L4 (1)
	L4 9L
118	40 9L
	40 81L
119	F5 12L
	42 12L
120	42 23L
	42 24L
121	F5 6F
	40 6F
122	22 8L
	F5 (C1)
123	36 115L
	F5 6L
124	42 6L
	L4 (1)
125	42 10L
	42 78L
126	42 79L
	F5 (V2)
127	42 12L
	42 23L
128	42 24L
	L5 3F
129	40 2F
	L4 (1)
130	40 3F
	L4 (C2)
131	40 (C2)
	40 9L
132	40 81L
	L5 3F
133	00 20F
	46 10L

LOCATION	ORDER
134	46 82L
	F5 25L
135	42 25L
	42 42L
136	42 46L
	L5 3F
137	L4 5L
	40 5L
138	L5 12L
	L4 (1-1)
139	46 12L
	F5 (C1)
140	26 3L
	F5 F
141	40 F
	L7 5F
142	50 F
	00 1F
143	40 5F
	26 67L
144	L5 (1)
	40 3F
145	L1 (N)
	40 F
146	40 5F
	L5 (V3)
147	42 119L
	42 120L
148	L5 (MS)
	L4 F
149	32 121L
	L7 F
150	40 (MS)
	F5 F
151	32 15(MTR)
	40 F

LOCATION	ORDER	NOTES
152	F5 119L	
	26 118L	
153	FF F	
	26 18(MTR)	FFOOO; matrix is singular, wh. sw. to begin new problem
	OOK	
0	L3 F	
	40 608F	
1	22 548F	
	50 F	
	26 L	
	26 1N	
	0052K	
52	00 F	
	00 66F	
53	L5 196F	
	40 349F	Initialize addresses for rest of program
54	L5 (V1)	
	42 176F	
55	42 194F	
	42 195F	
56	42 268F	
	42 279F	
57	L5 (V2)	
	42 210F	
58	42 211F	
	42 262F	
59	42 264F	
	42 280F	
60	42 281F	
	L5 81F	
61	40 246F	
	40 410F	
62	40 527F	
	40 238F	
63	L5 553F	
	L4 196F	

LOCATION	ORDER	NOTES PAGE 16 M 34
64	40 399F	
	L5 (1-1)	
65	46 312F	
	46 400F	
66	46 411F	
	46 239F	
67	46 247F	
	L5 (V3)	
68	42 200F	
	42 201F	
69	42 233F	
	40 159F	
70	L5 138F	
	40 521F	
71	40 546F	
	40 503F	
72	F5 (MS)	
	00 20F	
	0073K	
73	46 1L	
	L5 (+1)	Rescale L routine
74	10 F	
	00 1F	
75	40 6F	
	L5 (V1)	
76	42 12L	
	42 13L	
77	L5 3F	
	L4 12L	
78	40 4F	
	L5 S5	
79	L4 (MS)	
	40 2F	
80	50 S3	
	50 7L	Playback a row of L

LOCATION	ORDER	NOTES
81	26 (Y1)	
	00 S6	
82	00 1F	
	L3 2F	
83	36 17L	
	L7 2F	
84	00 20F	
	46 13L	
85	01 1F	
	L5 F	Rescale a row of L
86	10 F	
	40 F	
87	F5 12L	
	42 12L	
88	42 13L	
	L0 4F	
89	36 17L	
	22 12L	
90	L5 S3	
	40 S4	
91	L5 6F	
	40 S3	
92	J0 S3	
	50 19L	Record a row of L
93	26 (Y1)	
	00 S6	
94	00 1F	
	F5 5F	
95	32 31L	
	40 5F	
96	F5 5L	
	42 5L	
97	F5 3F	
	40 3F	
98	L4 8L	
	40 8L	

LOCATION		ORDER	NOTES
99		40 20L	
		L5 9L	
100		L4 (1-1)	
		46 9L	
101		46 21L	
		L5 17L	
102		L4 (1-1)	
		40 17L	
103		F5 18L	
		42 18L	
104		22 2L	
		41 (Z)	
105		L1 (MS)	
		40 S5	
		00K	
106	(IT)	L5 (C2)	
		40 18L	
107		40 77L	
		L1 (N)	
108		40 2F	
		L4 (1)	
109		40 1F	
		L5 87L	
110		L0 (1-1)	
		40 3F	
111		L0 (1-1)	
		L4 22L	
112		40 22L	
		L1 1F	
113		L4 88L	
		40 88L	
114		L5 70L	
		L0 2F	
115		40 87L	
		L5 3F	

LOCATION	ORDER	NOTES
116	L4 53L	
	40 53L	
117	42 16L	
	L1 2F	
118	40 3F	
	40 4F	
119	00 20F	
	46 19L	
120	46 78L	
	F5 (MS)	
121	00 20F	
	46 30L	
122	L1 (MS)	
	40 F	
123	50 S3	
	50 17L	Playback row of L
124	26 (Y1)	
	00 F	
125	00 F	
	L5 18L	
126	L0 91L	
	L4 364F	
127	40 5F	
	40 6F	
128	L1 S3	
	40 S3	
129	F5 1F	
	32 76L	
130	L5 22L	
	L0 (1)	
131	42 29L	
	42 34L	
132	42 57L	
	L5 86L	
133	40 37L	
	41 F	

LOCATION	ORDER	NOTES
134	41 7F	
	41 8F	
135	50 7F	
	L5 F	
136	10 F	
	00 1F	
137	26 35L	
	40 3F	
138	85 11F	
	00 S6	
139	40 9F	
	50 9F	Form $\ell_{jm}^{-1} \ell_{mk}$
140	L5 7F	
	74 F	
141	40 9F	
	S5 F	
142	40 7F	
	L5 9F	
143	10 1F	
	00 1F	
144	L4 F	
	40 F	
145	LL F	
	36 44L	
146	L5 F	
	10 1F	
147	40 F	
	F5 8F	
148	40 8F	
	L5 37L	
149	L4 (1-1)	
	46 37L	
150	F5 34L	
	42 34L	
151	L0 88L	
	32 64L	

LOCATION	ORDER
152	L5 5F
	40 32L
153	L4 3F
	40 5F
154	F5 3F
	22 31L
155	L5 F
	00 1F
156	40 F
	F5 8F
157	36 57L
	26 66L
158	L3 8F
	36 57L
159	L4 S5
	40 S5
160	L1 8F
	50 (1)
161	00 20F
	L4 30L
162	46 30L
	50 F
163	L1 F
	40 F
164	L5 89L
	L0 57L
165	32 74L
	L5 6F
166	L0 4F
	40 5F
167	40 6F
	L5 4F
168	L0 (1)
	40 3F
169	40 4F
	L5 57L

LOCATION	ORDER	NOTES
170	22 24L	
	50 7F	
171	L1 8F	
	LO (MS)	
172	40 8F	
	LL F	
173	36 49L	
	L7 8F	
174	00 20F	
	46 71L	
175	F5 29L	
	42 70L	
176	42 71L	
	L5 S3	
177	10 F	
	40 F	
178	F5 70L	
	LO 87L	
179	36 52L	
	F5 70L	
180	22 69L	
	L5 22L	
181	LO (1-1)	
	40 22L	
182	JO S3	
	50 76L	Store row of L^{-1} on drum
183	26 (Y1)	
	00 F	
184	00 F	
	F5 1F	
185	36 92L	
	40 1F	
186	L5 18L	
	L4 2F	
187	40 18L	
	40 77L	

LOCATION	ORDER
188	F5 2F
	40 2F
189	L5 88L
	L0 (1)
190	42 88L
	L1 (1-1)
191	26 10L
	50 F
192	10 1F
	00 1F
193	00 S9
	00 S9
194	L5 7F
	74 S3
195	L1 F
	40 S3
196	00 F
	00 S6
197	26 (Y1)
	00 3F
198	L1 (MS)
	40 3F
199	L1 (N)
	40 2F
200	L5 (MS)
	L4 S5
201	32 96L
	L7 S5
202	40 (MS)
	F5 94L
203	42 94L
	42 95L
204	F5 2F
	32 99L
205	22 93L
	L1 (N)

LOCATION		ORDER	NOTES	PAGE 24	M 34
206		40 2F			
		40 5F			
207		L1 (MS)			
		L6 3F			
208		36 233F			
		40 F			
209		L7 F			
		00 20F			
210		46 105L			
		L5 S4			
211		10 F			
		40 S4			
212		F5 2F			
		36 233F			
213		40 2F			
		F5 104L			
214		42 104E			
		42 105L			
215		22 104L			
		50 F			
		00K			
0		L3 F			
		40 609F			
1		22 551F			
		50 F			
		26 L			
		26 1N			
		00 216K			
216	(R)	40 1F	Reciprocal routine		
		K5 F			
217		42 16L			
		43 13L			
218		41 F			
		50 F			
219		26 5L			
		LJ F			

LOCATION	ORDER	NOTES
220	00 1F	
	40 1F	
221	F5 13L	
	42 13L	
222	LJ 1F	
	40 F	
223	32 3L	
	LJ F	
224	40 F	
	66 1F	
225	11 1F	
	L5 F	
226	36 12L	
	K1 F	
227	50 L	
	26 13L	
228	S5 F	
	50 L	
229	10 38F	
	00 F	
230	10 1F	
	40 1F	
231	S5 F	
	40 F	
232	L5 1F	
	22 F	
	00K	
233	L5 (MS)	
	L4 S5	Rescale L^{-1} routine
234	L4 (1)	
	42 7L	
235	L5 (V1)	
	42 6L	
236	42 8L	
	22 4L	

LOCATION	ORDER	NOTES PAGE 26 M 34
237	50 S3	
	50 4L	Playback row of L^{-1}
238	26 (Y1)	
	00 S6	
239	00 1F	
	50 F	
240	7J (-)	
	10 F	
241	00 1F	
	40 F	Rescale a row of L^{-1}
242	F5 8L	
	42 6L	
243	42 8L	
	L0 24L	
244	32 12L	
	22 6L	
245	J0 S3	
	50 12L	
246	26 (Y1)	
	00 S6	Record row of L^{-1}
247	00 1F	
	F5 5F	
248	40 5F	
	36 (D)	
249	F5 24L	
	42 24L	
250	F5 L	
	42 L	
251	F5 23L	
	40 23L	
252	L4 5L	
	40 5L	
253	40 13L	
	L5 6L	
254	L4 (1-1)	
	46 6L	

LOCATION		ORDER	NOTES
255		46 14L	
		26 L	
256		00 F	
		00 1F	
257		00 1F	
		40 1S3	
		00K	
258	(D)	L5 (N)	
		L4 10L	Form D^{-1} routine
259		40 2F	
		L4 33L	
260		40 3F	
		L3 (MS)	
261		L4 (34)	
		32 4L	
262		26 34L	
		L5 S4	
263		32 5L	
		50 5L	
264		26 (R)	
		40 S4	Form $d_{jj}{}^{-1}$
265		01 1F	
		L6 1F	
266		40 1F	
		09 1F	
267		L6 1F	
		36 35L	
268		L5 F	
		40 S3	
269		L7 5F	
		L2 1F	
270		32 13L	
		L7 1F	
271		40 5F	
		F5 10L	

LOCATION	ORDER	NOTES
272	42 10L	
	LO 2F	
273	32 17L	
	F5 4L	
274	42 4L	
	42 6L	
275	22 4L	
	L5 (+)	
276	40 2F	
	L5 (-)	
277	40 6F	
	L7 5F	
278	LO 2F	
	36 28L	
279	50 6F	
	7J S3	
280	50 6F	
	74 S4	
281	00 39F	
	40 S4	
282	F5 21L	
	42 21L	
283	LO 3F	
	36 (MP)	
284	F5 22L	
	42 22L	
285	42 23L	
	26 21L	
286	F5 (Z)	
	40 (Z)	
287	50 (+)	
	75 2F	
288	S5 F	
	40 2F	
289	50 (-)	
	7J 6F	

LOCATION		ORDER	NOTES PAGE 29 M 34
290		26 19L	
		50 F	
291		5S 6F	
		3J F	
292		FF 1F	
		26 18(MTR)	FF001; matrix over-scaled, wh. sw. to be-
293		FF 2F	gin new problem
		26 18(MTR)	FF002; diagonal element too small, wh. sw.
		00K	to begin new problem
294	(MP)	L1 (N)	
		40 1F	
295		41 7F	
		L5 (1)	
296		40 6F	
		F5 (1)	
297		40 5F	
		L5 (V2)	
298		42 4F	
		42 18L	
299		L5 55L	
		L4 (D1)	
300		40 17L	
		L5 55L	
301		L4 56L	
		40 47L	
302		41 3F	
		L5 6F	
303		40 2F	
		L5 (V3)	
304		42 12L	
		L5 1F	
305		40 F	
		L5 (1)	
306		40 8F	
		41 F	

LOCATION	ORDER	NOTES
307	F5 F	
	40 F	
308	32 16L	
	F5 12L	
309	42 12L	
	22 12L	
310	50 S3	
	50 16L	
311	26 (U)	
	00 S6	Playback a row of submatrix of L^{-1}
312	00 1F	
	50 F	
313	7J S3	
	40 9F	Form $d_m^{-1} \ell_{mk}^{-1}$
314	L1 8F	
	40 F	
315	L5 (V3)	
	40 24L	
316	L5 (V1)	
	42 23L	
317	50 9F	
	7J F	
318	L4 F	
	40 F	$\sum_{m=1}^{n} \ell_{jm}^{-1} d_m^{-1} \ell_{mk}^{-1}$
319	F5 F	
	32 35L	
320	40 F	
	L5 24L	
321	L4 (1-1)	
	40 24L	
322	F5 23L	
	22 22L	
323	F5 1F	
	36 (OUT)	
324	40 1F	
	L5 5F	

LOCATION	ORDER	NOTES
325	40 6F	
	L4 (1)	
326	40 5F	
	L4 55L	
327	40 55L	
	L5 (1-1)	
328	46 18L	
	F5 4F	
329	26 4L	
	F5 3F	
330	40 3F	
	L4 1F	
331	32 42L	
	F5 8F	
332	40 8F	
	F5 2F	
333	40 2F	
	L4 17L	
334	40 17L	
	L5 18L	
335	L4 (1-1)	
	40 18L	
336	22 16L	
	L5 1F	
337	40 F	
	L5 6F	
338	40 8F	
	L5 (V3)	
339	42 46L	
	42 48L	
340	42 49L	
	L5 F	
341	86 11F	
	00 F	Store a_{jk}^{-1} on drum
342	L7 7F	
	L2 F	

LOCATION		ORDER	NOTES
343		32 50L	
		L7 F	
344		40 7F	
		F5 F	
345		36 29L	
		40 F	
346		F5 8F	
		40 8F	
347		L4 47L	
		40 47L	
348		F5 46L	
		26 45L	
349		00 F	
		00 S6	
350		86 11F	
		00 F / 00 K	
351	(U)	K5 F	
		42 2L	Playback submatrix of L^{-1} or A^{-1} routine
352		L4 (1)	
		42 4L	
353		42 10L	
		L5 F	
354		L0 (D1)	
		L4 13L	
355		40 7L	
		L5 F	
356		10 20F	
		40 9F	
357		S5 F	
		42 8L	
358		85 11F	
		00 F	
359		32 8L	
		40 F	
360		F5 F	
		40 F	

LOCATION		ORDER
361		L0 9F
		32 F
362		F5 7L
		40 7L
363		F5 8L
		22 6L
364		85 11F
		00 F
		00K
365	(OUT)	F5 (1)
		L4 (Z)
366		40 (Z)
		41 1F
367		L5 (+1)
		40 2F
368		L3 (MS)
		36 13L
369		40 F
		LL 7F
370		32 9L
		F5 (Z)
371		40 (Z)
		50 (-)
372		7J 2F
		40 2F
373		50 (-)
		75 7F
374		22 10L
		50 1F
375		L5 7F
		00 1F
376		40 7F
		F5 F
377		36 13L
		26 4L

LOCATION	ORDER	NOTES
378	L5 (1)	
	40 3F	
379	L7 7F	
	LO (-)	
380	36 21L	
	50 (+)	
381	75 3F	
	S5 F	
382	40 3F	
	50 (+)	
383	77 7F	
	S5 F	
384	40 7F	
	F5 1F	
385	40 1F	
	26 14L	
386	L5 1F	
	LO (Z)	
387	36 27L	
	40 F	
388	L5 (-)	
	40 (Z)	Form point indicator
389	F5 F	
	36 27L	
390	40 F	
	50 (-)	
391	7J (Z)	
	22 23L	
392	L1 (N)	
	40 4F	
393	F5 (MS)	
	00 20F	
394	46 40L	
	46 57L	
395	46 69L	
	L5 (1)	

LOCATION	ORDER	NOTES
396	40 5F	
	L4 69L	
397	40 6F	
	41 F	
398	50 S3	
	50 33L	
399	26 (U)	
	00 S6	Playback a row of A^{-1}
400	00 1F	
	L3 1F	
401	36 54L	
	L5 (V1)	
402	42 38L	
	42 40L	
403	50 3F	
	75 F	
404	00 39F	
	10 1F	Rescale a row of A^{-1}
405	00 F	
	40 F	
406	F5 40L	
	L0 6F	
407	32 44L	
	F5 40L	
408	26 37L	
	50 F	
409	J0 S3	
	50 44L	Record a row of A^{-1}
410	26 (Y1)	
	00 S6	
411	00 1F	
	F5 4F	
412	40 4F	
	32 60L	
413	L5 35L	
	L4 (1-1)	

LOCATION	ORDER	NOTES
414	46 35L	
	46 46L	
415	F5 5F	
	L4 34L	
416	40 34L	
	F5 5F	
417	L4 45L	
	40 45L	
418	F5 5F	
	26 31L	
419	L5 (V1)	
	42 55L	
420	42 57L	
	50 F	
421	7J 2F	
	10 1F	
422	00 F	
	40 F	
423	F5 57L	
	L0 6F	
424	32 44L	
	F5 57L	
425	22 54L	
	F5 (P)	
426	00 7F	
	40 F	Form counter for number of elements per line
427	L5 (P)	
	00 20F	
428	46 72L	
	L5 52F	
429	66 F	
	01 7F	
430	40 3F	
	L3 (O)	Output option test:
431	36 (TP)	
	L4 (1)	Jump to triangle print

LOCATION	ORDER	NOTES
432	36 (SP)	
	L4 (1)	Jump to square print
433	36 (DP)	
	26 18(MTR)	Jump to diagonal print
434	00 F	
	40 S3	
435	92 834F	
	92 147F	Punch J, punch 5 cr/lf
436	92 515F	
	L5 (Z)	Punch delay
437	50 F	
	50 72L	Print point indicator
438	26 (P16)	
	92 147F	Punch 5 cr/lf
439	92 515F	
	26 18(MTR)	Punch delay
440	(P16)00K	
	00K	
496	(SP) L5 72(OUT)	
	46 8L	
497	L1 (N)	
	40 5F	
498	F5 4F	
	40 4F	
499	40 6F	
	L1 (N)	
500	40 7F	
	L1 3F	
501	40 8F	
	L1 4F	
502	40 9F	
	36 7L	
503	85 11F	
	00 S6	
504	50 F	
	50 8L	Print a_{jk}^{-1}

LOCATION	ORDER	NOTES
505	26 (P16)	
	F5 7F	
506	32 19L	
	40 7F	
507	F5 8F	
	32 17L	
508	40 8F	
	F5 9F	
509	32 15L	
	40 9F	
510	F5 7L	
	40 7L	
511	26 7L	
	F5 6F	
512	40 6F	
	L4 7L	
513	22 14L	
	92 131F	Punch cr/lf
514	92 515F	
	L1 3F	Delay
515	26 12L	
	92 770F	Punch N
516	F5 5F	
	36 70(OUT)	
517	40 5F	
	F5 4F	
518	L4 25L	
	40 25L	
519	40 7L	
	92 131F	Punch cr/lf
520	92 515F	
	26 2L	Delay
521	85 11F	
	00 S6	

LOCATION		ORDER	NOTES
		00K	
522	(TP)	L5 72 (OUT)	
		46 10L	
523		L1 (N)	
		40 5F	
524		F5 4F	
		40 4F	
525		00 20F	
		46 6L	
526		50 S3	
		50 4L	Playback row of A^{-1}
527		26 (Y1)	
		00 S6	
528		00 1F	
		L5 (V1)	
529		42 9L	
		L1 4F	
530		40 6F	
		L1 3F	
531		40 7F	
		L5 F	
532		50 F	
		50 10L	Print a_{jk}^{-1}
533		26 (P16)	
		F5 6F	
534		36 17L	
		40 6F	
535		F5 9L	
		42 9L	
536		F5 7F	
		32 15L	
537		26 9L	
		92 131F	
538		92 515F	
		22 8L	

LOCATION	ORDER	NOTES
539	F5 5F	
	36 70(OUT)	
540	40 5F	
	F5 4F	
541	L4 5L	
	40 5L	
542	92 135F	
	92 515F	Punch cr/lf, delay
543	26 2L	
	50 F	
	00K	
544 (DP)	L5 72(OUT)	
	46 3L	
545	L1 (N)	
	40 4F	
546	85 11F	
	00 S6	
547	50 F	
	50 3L	Print a_{jj}^{-1}
548	26 (P16)	
	F5 27F	
549	40 27F	
	L4 2L	
550	40 2L	
	92 131F	Punch cr/lf
551	92 515F	
	F5 4F	Delay
552	32 70(OUT)	
	22 1L	
553 (D1)	26 (U)	
	00 F	

LOCATION	ORDER	NOTES PAGE 41 M34
	00610K	
0	L3 F	
	30 554F	
1	FF 3F	
	22 554F	FF003; master read incorrectly, reread.
2	LK 3334F	
	OK 0604F	
	26 L	
	26 1N	