UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

AUXILIARY
LIBRARY ROUTINE V 12 - 304

TITLE: Ordinary Bessel Functions (SADOI Only)

TYPE: Closed

NUMBER OF WORDS: 242

TEMPORARY STORAGE: 0 - 10 inclusive

ACCURACY: At least 7 decimal places

DURATION: Approximately 170 m.sec.

DESCRIPTION: This routine calculates the Ordinary Bessel Functions which are defined as follows:

$$J_n(z) = \sum_{m=0}^{\infty} \frac{(-1)^m z^{n+2m}}{2^{n+2m}\, m!\, (m+n)!}$$

$$N_n(z) = \frac{J_n(z) \cos n\pi - J_{-n}(z)}{\sin n\pi}$$

where n is an integer and is confined to M = 0, 1 in this routine. These functions are solutions to Bessel's equation:

$$z^2 \frac{d^2w}{dz^2} + z \frac{dw}{dz} + (z^2 - n^2)\, w = 0$$

The accuracy of the routine depends upon the accuracy of the argument. For this reason, a double precision argument may be used. In cases where the argument, scaled by a suitable scaling factor, has sufficient accuracy in single precision, a single precision argument may be used.
The true value of the argument must be greater than .000005. Values smaller than this will give incorrect results.
To use a double precision argument, where the argument is scaled by 2^{-M}, store the most significant part in location OF and the least significant part in the accumulator and enter with

	50 MF
p	50 pF
p+1	26 ()

where $M \geq 0$ but $M \leq 39$.

If a single precision argument is to be used, the accumulator must be clear upon entry. One form for entry may be

	50 MF
p	51 pF
p+1	26 ()

Again the MSP must be in location 0F.

The routine will leave the following information in the locations shown:

in location	7F	$J_0(x)$	
" "	8F	$1/16N_0(x)$	if $x < 3$
" "	8F	$N_0(x)$	if $x \geq 3$
" "	9F	$J_1(x)$	
" "	10F	$XN_1(x)$	if $x < 3$
" "	10F	$N_1(x)$	if $x \geq 3$

NOTE: Notice the two forms for $N_0(x)$ and $N_1(x)$ depending on the size of the argument.

The aim of the double precision argument is to allow the use of a wide range of arguments, all of which may be scaled by the same scaling factor. This means, for example, that if a range of arguments is from .00005 to 2^{12}, they may all be scaled by 2^{13}. The routine rescales each value down to the minimum value needed for each. For small arguments scaled by large factors, a double precision argument is needed to insure the accuracy of the routine.

This routine contains the subroutines; (S3) Natural Logarithm, (R1) Square Root, (T5) Sine-Cosine. These may be used by other parts of the program and may be entered in the usual manner since they are referred to in this routine as symbolic addresses.

The methods used to obtain these functions may be found in "Mathematical Tables and Aids to Computation" 1954, Vol. 8, pp. 240-241.

DATE June 15, 1960
PROGRAMMED BY Marvin Harding
APPROVED BY J N Snyder

nj

LOCATION	ORDER	NOTES PAGE 1 V 12
	00K(V12)	
0	40 1F	
	41 132L	
1	K5 F	
	42 121L	Plant link
2	10 20F	
	42 132L	
3	41 133L	
	26 197L	
4	L5 F	
	00 1F	Rescale
5	36 6L	
	26 9L	Argument
6	40 F	
	F5 133L	
7	40 133L	
	L0 132L	
8	36 9L	
	26 4L	
9	L5 F	
	40 5F	Arg. x $2^{-M'}$
10	L5 132L	
	L0 133L	
11	40 6F	
	L3 6F	
12	32 24L	$0 < \text{Arg} < 1$
	40 F	
13	F5 F	
	36 26L	$1 < \text{Arg} < 2$
14	40 F	
	F5 F	
15	36 16L	$2 \leq \text{Arg} < 4$
	26 74L	$\text{Arg} > 4$

LOCATION	ORDER	NOTES PAGE 2 V 12
16	L5 5F	Test for size
	L0 122L	of Argument
17	36 18L	
	26 28L	2 < Arg < 3
18	40 F	
	L3 F	
19	36 20L	Arg = 3
	26 74L	Arg > 3
20	L5 123L	
	40 7F	
21	L5 124L	Arg = 3
	40 8F	Place value of
22	L5 125L	
	40 9F	functions in
23	L5 126L	
	40 10F	locations
24	22 121L	
	L5 5F	
25	10 2F	
	26 27L	Prepare for
26	L5 5F	
	10 1F	small arg
27	40 5F	
	50 141L	
28	L5 5F	
	50 28L	
29	26 (S3)	
	10 6F	1/16 $\ln$ x/2
30	75 130L	
	00 2F	
31	L0 127L	
	40 131L	
32	50 128L	
	75 5F	

LOCATION	ORDER	NOTES PAGE 3 V 12
33	00 2F	Arg/3
	40 F	
34	L5 27L	
	42 67L	Preset for
35	L5 98L	J_0(small)
	46 70L	
36	26 63L	
	L5 196L	
37	L4 140L	
	00 2F	
38	40 7F	J_0(small)
	L5 73L	
39	42 67L	
	L5 101L	
40	46 70L	Preset for
	26 63L	N_0(small)
41	50 49L	
	L5 196L	
42	L4 147L	
	50 190L	
43	40 8F	
	50 131L	
44	75 129L	
	40 3F	
45	50 3F	
	75 7F	
46	L4 8F	
	40 8F	1/16 N_0(small)
47	L5 66L	
	42 67L	Preset for
48	L5 41L	J_1(small)
	46 70L	
49	26 63L	
	L5 196L	

LOCATION	ORDER	NOTES PAGE 4 V 12
50	L4 154L	
	40 2F	
51	50 2F	
	75 5F	
52	00 2F	
	40 9F	J (small)
53	L5 62L	
	42 67L	Preset for
54	L5 107L	N_1 (small)
	46 70L	
55	26 63L	
	L5 196L	
56	L4 161L	
	40 10F	
57	50 88L	
	50 3F	
58	75 9F	
	40 3F	
59	50 3F	
	75 5F	
60	00 2F	
	L4 10F	
61	00 4F	
	40 10F	$XN_1(x)$ small
62	22 121L	Return to master
	50 162L	
63	50 F	
	7J F	
64	40 1F	
	40 2F	
65	41 196L	
	L5 135L	
66	40 136L	
	50 155L	
67	50 2F	
	75 F	Form sum

LOCATION	ORDER	NOTES PAGE 5 V 12
68	L4 196L	of terms
	40 196L	
69	F5 136L	
	40 136L	
70	32 F	
	50 1F	
71	75 2F	
	40 2F	
72	F5 67L	
	42 67L	
73	26 67L	
	50 148L	
74	L5 6F	
	42 79L	
75	42 95L	
	42 99L	Prepare for
76	L5 139L	
	46 111L	large Arg.
77	L5 138L	
	42 90L	
78	50 134L	
	L5 183L	
79	L5 122L	
	10 F	
80	66 5F	
	00 2F	
81	S5 F	
	40 3F	
82	75 128L	
	50 82L	
83	26 (R1)	
	40 4F	$1/\sqrt{x}$
84	L5 87L	
	42 67L	

LOCATION	ORDER	NOTES PAGE 6 V 12
85	L5 57L	
	46 70L	
86	L5 3F	
	40 1F	
87	40 2F	
	50 169L	
88	26 65L	
	50 176L	
89	L5 196L	
	L4 168L	
90	40 10F	
	L5 F	Prepare for
91	42 67L	
	L5 117L	large arguments
92	46 70L	
	L5 3F	
93	40 1F	
	40 2F	
94	26 65L	
	L5 196L	
95	L4 175L	
	10 F	
96	40 F	
	L5 5F	
97	L0 F	
	40 F	
98	50 36L	
	50 F	
99	75 137L	
	00 F	
100	40 9F	
	L4 154L	
101	50 41L	
	50 101L	

LOCATION	ORDER	NOTES PAGE 7 V 12
102	26 (T5)	
	40 F	cos (Arg - ϕ)
103	50 10F	
	75 4F	
104	40 6F	
	50 6F	
105	75 F	
	00 1F	
106	40 7F	J_0(large)
	L5 9F	or J_1(large)
107	50 55L	
	50 107L	
108	26 (T5)	(T5)
	40 F	sin (Arg - ϕ)
109	50 F	
	75 6F	
110	00 1F	
	40 8F	N_0(large)
111	22 F	
	L5 138L	or N_1(large)
112	46 111L	
	L5 139L	
113	42 90L	
	L5 78L	
114	42 67L	
	L5 7F	
115	40 132L	
	L5 8F	
116	40 133L	
	26 85L	
117	50 94L	
	L1 7F	
118	40 10F	
	L5 8F	

LOCATION	ORDER	NOTES PAGE 8 V 12
119	40 9F	
	L5 132L	
120	40 7F	
	L5 133L	
121	40 8F	
	22 F	Return to master
122	00 F	
	00 7500 0000 0000 J	
123	80 F	
	00 7399 4804 5100 J	
124	00 F	
	00 3768 5000 0000 J	
125	00 F	
	00 3390 5895 8530 J	
126	00 F	
	00 3246 7440 0000 J	
127	80 F	
	00 9566 7830 1220 J	
128	00 F	
	00 3333 3333 3333 J	
129	00 F	
	00 6366 1977 2368 J	
130	00 F	
	00 6931 4718 0560 J	
131	00 F	
	00 F	
132	00 F	
	00 F	
133	00 F	
	00 F	
134	00 F	
	00 F	
135	LL 4095F	
	LL 4090F	

LOCATION	ORDER	NOTES PAGE 9 V 12
136	00 F	
	00 F	
137	00 F	
	00 3183 0988 6184 J	
138	00 117L	
	00 88L	
139	00 111L	
	00 42L	
140	00 F	
	00 2500 0000 0000 J	
141	80 F	
	00 4375 0007 5000 J	Coefficients
142	00 F	
	00 3164 0520 0000 J	for
143	80 F	
	00 9209 0335 0000 J	J_0(small)
144	00 F	
	00 111 1975 0000 J	Scaled by
145	80 F	
	00 9990 1390 0000 J	2^{-2}
146	00 F	
	00 5250 0000 J	
147	00 F	
	00 2296 6681 870 J	
148	00 F	
	00 378 4960 3750 J	Coefficients
149	80 F	
	00 9535 3101 0000 J	for
150	00 F	
	00 158 1257 3120 J	N_0(small)
151	80 F	
	00 9973 3675 1000 J	scaled by 2^{-4}
152	00 F	
	00 2 6744 7500 J	

LOCATION	ORDER	NOTES PAGE 10 V 12
153	80 F	
	00 9998 4471 2500 J	
154	00 F	
	00 5000 0000 0000 J	
155	80 F	
	00 4375 0015 0000 J	Coefficients
156	00 F	
	00 2109 3573 0000 J	for
157	80 F	J_1(small)
	00 9604 5711 0000 J	
158	00 F	
	00 44 3319 0000 J	unscaled
159	80 F	
	00 9996 8239 0000 J	
160	00 F	
	00 1109 0000 J	
161	80 F	
	00 9602 1126 4500 J	
162	00 F	
	00 138 2556 8750 J	Coefficients
163	00 F	
	00 1355 1693 1200 J	for
164	80 F	
	00 9177 1983 1300 J	N_1(small)
165	00 F	scaled by
	00 195 2469 3750 J	2^{-4}
166	80 F	
	00 9974 9390 0000 J	
167	00 F	
	00 1 7420 6250 J	
168	00 F	
	00 7978 8456 0000 J	
169	80 F	
	00 9999 9923 0000 J	Coefficients

LOCATION	ORDER	NOTES PAGE 11 V 12
170	80 F	
	00 9944 7260 0000 J	for
171	80 F	f_0
	00 9999 0488 0000 J	
172	00 F	
	00 13 7237 0000 J	unscaled
173	80 F	
	00 9992 7195 0000 J	
174	00 F	
	00 14476 0000 J	
175	00 F	
	00 7853 9816 0000 J	
176	00 F	
	00 416 6397 0000 J	Coefficients
177	00 F	
	00 3954 0000 J	for
178	80 F	ϕ_0
	00 9973 7427 0000 J	unscaled
179	00 F	
	00 5 4125 0000 J	
180	00 F	
	00 2 9333 0000 J	
181	80 F	
	00 9998 6442 0000 J	
182	00 F	
	00 7978 8456 0000 J	
183	00 F	
	00 156 0000 J	Coefficients
184	00 F	for
	00 165 9667 0000 J	
185	00 F	f_1
	00 1 7105 0000 J	unscaled
186	80 F	
	00 9975 0489 0000 J	

LOCATION	ORDER	NOTES PAGE 12 V 12
187	00 F	
	00 11 3653 0000 J	
188	80 F	
	00 9997 9670 0000 J	
189	00 F	
	00 7853 9816 0000 J	
190	80 F	
	00 8750 0388 0000 J	
191	80 F	
	00 9999 4350 0000 J	Coefficients
192	00 F	for
	00 63 7879 0000 J	
193	80 F	ϕ_1
	00 9992 5652 0000 J	unscaled
194	80 F	
	00 9992 0176 0000 J	
195	00 F	
	00 2 9166 0000 J	
196	00 F	
	00 F	
197	L3 132L	
	36 9L	
198	50 1F	
	26 4L	
	00K(S3)	199-212
	00K(R1)	213-221
	00K(T5)	222-242

Add these tapes in this order.