UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER LABORATORY

NEW COMPUTER LIBRARY ROUTINE B1-SIN-29

TITLE: Sine-Cosine

TYPE: closed, multiple entry, relocatable, mnemonic

LENGTH: 13 words

TEMPORARY STORAGE: 1 word at a location specified by the symbol "common" (to be defined by the programmer) and F2

DURATION: 100 microseconds (January, 1963)

FAST REGISTERS CHANGED: F2

SUBROUTINES USED: none

ACCURACY: absolute error $< 2^{-43}$

PARAMETERS: link in M3

ENTRIES: Enter with $x = \theta/\pi$ in the accumulator. Suppose N is the address of the first word of this routine.

1) JSB3,2,N (or JSB3,,sin)

$\sin \pi x$ is placed in A and OV is cleared.

2) JSB3,,N+1 (or JSB3,,cos)

$\cos \pi x$ is placed in A and OV is cleared.

Note: In general the result is unnormalized.

METHOD: Entrance at "sin" places $x - 1/2$ in A and computes $\cos \pi (x - 1/2) = \sin \pi x$. $x \pmod 2$ is formed at "cos" and then $x := 1/2 - |x|$. The identity $\sin (\pi/2 - |\pi x|) = \cos \pi x$ is used at this point.

$\sin \pi x$ is now computed using the Tchebyscheff polynomial approximation of degree 13 to the Taylor series expansion of $\sin (\pi/2x)$, $-1/2 \leq x \leq 1/2$. The polynomial,

$$\sin x \approx \sum_{k=0}^{6} c_k x^{2k+1},$$

is evaluated by the standard technique. The coefficients were calculated on Illiac II, starting with $\sin y \sim y - y3/3! + \ldots - y19/19!$

REFERENCE: Hildebrand, F. B., "Introduction to Numerical Analysis," McGraw-Hill, New York (1956).

DATE:	January 28, 1963
PROGRAMMED BY:	Robert Longes
APPROVED BY:	

SINE-COSINE

	FLD	
0	DEC-6,cos+5	modifier constants
	sinSUB10,3,2048	compute sine
1	cosSTF0,3	compute cosine; x = x (mod2)
	SFR6,2,common	save F6
	JDC10,1,cos+1	clear OV
2	DAV10,3,2048	|x| - 1/2 → A
	STN2,3	x: = 1/2 - |x|
3	MPY0,3	x^2 → A
	LFR6,2,cos-1	set M8 and M9, i: = -6
	STR0,3	store x^2 in F0
4	CAD9,1	C_7 → A
	MPY0,3	$*x^2$
	ADD9,1	$+C_{-i}$
	CJF8,1	is i = 0
5	LFR6,2,common	restore F6
	MPY2,3	*x
	JLN3	→ exit
6	OCT05517,06174,04521,15173	c_7 = 3.14159265358859
7	OCT14165,15472,10711,10575	c_6 = -5.16771277987331
8	OCT02501,15532,14354,01577	c_5 = 2.55016405224337
9	OCT13151,06467,17623,06400	c_4 = -.59926438371110
10	OCT05063,05706,06336,13601	c_3 = .8214445992426$_{10}$-1
11	OCT15525,01030,14735,04602	c_2 = -.736295450191$_{10}$-2
12	OCT06220,17665,04210,14201	c_1 = .44617418582$_{10}$-3